



Performance modelling of health-care service delivery in Adekunle Ajasin University, Akungba-Akoko,(AAUA) Nigeria using Queuing theory

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Abstract

Keywords:

Performance,
Health-care,
Service delivery,
Queuing theory,
Simulation,
M/M/S

Queuing theory is the mathematical approach to the analysis of congestion and delays of waiting in line. It is generally considered a branch of operations research and often used in decisions making about resources needed to provide a service. It is very applicable to health care setting where they have patients waiting in line to be served. The purpose of this study determines the waiting, arrival and service times of patients at AAUA health-centre, to model a suitable queuing system and use simulation technique to validate the model. It employed analytical and simulation methods to develop the suitable model. The collection of waiting time for this study was based on the arrival rate and service rate of patients at the outpatient Centre. The data was calculated and analyzed using Microsoft Excel. Based on the analyzed data while the queuing system of the patients' current situation was modeled and simulated using the Python programming language. The result obtained from the simulation model showed that the mean arrival rate of patients on Friday week1 (i.e. $5.33 > 5.625$ ($\lambda > \mu$)). The analysis of the entire system of the AAUA health centre showed that queue length increases when the system is very busy. This work therefore evaluated and predicted the system performance of AAUA Health-Centre in terms of service delivery and propose solutions on needed resources to improve the quality of service offered to the patients visiting the health centre.

Accepted:7 June 2019

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INTRODUCTION

It is a universally acknowledged goal that a healthcare system should treat its patients and especially those in need of critical care – in a timely manner. However, this is often not achieved in practice, particularly in public healthcare systems that suffer from high patient demand and limited resources (Au-Yeung *et al.*, 2009).

The service facilities whose patients vary generally in capacity and size, from small outpatient clinic to large, urban hospital to referral hospital. Regardless these differences, healthcare processes can be

categorized based on how patient arrive, wait for service, obtain service, and then depart. The healthcare processes also vary in complexity and extent, but they all deal with a set of both medical and non-medical activities and procedures that the patient must experience before getting the desired treatment. The servers in hospital queuing systems are the trained staff and equipment required for specific activities and procedures (Obamiro, 2013).

Queues are ubiquitous, particularly in health care delivery system. At the same time, queues are undesirable because delay in receiving needed service can cause prolonged discomfort and

economic loss when patient are unable to work and possible worsening of their medical condition that can increase subsequent diagnosis and or treatment to the extent that death occurs while patients waits (Gupta, 2013).

Adekunle Ajasin University, Akungba-Akoko (AAUA) is a Public University located in South-Western Nigeria. It has a health-centre that services the health-care needs of not only the staff and students of the University but also that of the immediate University community. As a result the population at the centre has always contributed to the queues problem in the health centre. Therefore, this work modeled the performance of the health centre in terms of service delivery using Queuing theory, with a view to assessing the current state and suggesting measures to improve its performance

LITERATURE REVIEW

Queuing theory deals with the study of queues which abound in practical situations and arise so long as arrival rate of any system is faster than the system can handle (Worthington, 2009). Queuing theory is applicable to any situation in general life ranging from cars arriving at filling station for fuel, customers arriving at a bank for various services, and in Health-care setting (Adedayo *et al.*,2006).

Queuing theory can be applied to the analysis of waiting lines in health-care setting; queuing analysis can be used as short-term measure or for facilities and resources planning. According to Obamiro (2006), queuing phenomenon comprises of the following basic characteristics: (1) arrival characteristics; (2) the queue or the physical line itself; (3) the number of servers or service channels; (4) queue discipline; (5) service mechanism; (6) The capacity of the system; (7) departure.

Agnihotri and Taylor (1991) modeled appointment systems and waiting list management in outpatient clinics. Tucker *et al.*, (1999) used an M/M/1 model to assess operating room staffing needs for night shifts.

Peinus *et al.*,(2000) used a non- preemptive priority queuing model to assess the average waiting times of emergent and non-emergent patients for computed tomography scans.

Nosek and Wilson (2001) used queuing theory in pharmacy applications with particular attention to improving patient’s satisfaction. Patients’ satisfaction is improved by predicting and reducing waiting time and adjusting staffing plan.

Vasanawala and Desser (2005) used Poisson probabilities to predict the required number of reserved slots (on weekly basis) for emergency radiology given that 95% of the requests are accommodated.

Olorunsola *et al.*,(2014) used the M/M/C Model to model the flow of in-patient in hospital; determine the optimal bed count and its performance measure. Aziati *et al.*,(2018) used descriptive analytical and simulation method to develop a suitable model. The collection of waiting time for this study was based on the arrival rate and service rate of patients at the outpatient counter. The data calculated and analyzed using Microsoft Excel. Based on the analyzed data, the queuing system of the patient current situation was modeled and simulated using the ARENA software.

METHOD AND MATERIALS

Data Collection

Visits to various sections of the health-centre after approval and clearance from the administrator of the hospital were conducted. The research instrument adopted for data collection was observation. A stop watch was used to calculate the number of minutes spent by each patient from the reception section where patients arrive and collect their hospital cards or register to the last section (the consulting room section). Data on the arrival time, waiting time and service time of each patient was collected on Weekdays (Mondays through Fridays) for three (3) weeks for four (4) hours daily between 8:00am to 12 noon.

Table 1: Data collected for concluding week

WEEK 3		
MONDAY		
ARRIVAL TIME	WAITING TIME	SERVICE TIME
8:30	10Min	5Min
8:50	5Min	3Min
9:25	5Min	3Min
10:15	2Min	5Min
10:56	3Min	5Min
TUESDAY		
ARRIVAL TIME	WAITING TIME	SERVICE TIME
9:00	15Min	5Min
9:50	5Min	3Min
10:15	3Min	5Min
10:55	4Min	4Min
11:18	2Min	3Min
11:53	2Min	2Min
WEDNESDAY		
ARRIVAL TIME	WAITING TIME	SERVICE TIME
8:50	10Min	3Min
9:55	2Min	5Min
10:25	10Min	10Min
10:25	2Min	5Min

THURSDAY		
ARRIVAL TIME	WAITING TIME	SERVICE TIME
9:15	15Min	9Min
9:15	24Min	5Min
10:50	2Min	5Min
11:40	3Min	5Min
FRIDAY		
ARRIVAL TIME	WAITING TIME	SERVICE TIME
8:25	10Min	5Min
8:45	7Min	10Min
8:45	17Min	5Min
9:20	5Min	5Min
10:15	4Min	5Min
10:15	3Min	2Min
11:25	3Min	3Min

Formulation of Analytical Model

The queuing model adopted for this work is the M/M/S model or Erlang C model which is the most commonly used queuing model (Green *et al.*, 2006). The assumptions for this model are that there is a single queue with unlimited waiting room that are going to be serviced by S identical servers (doctors). Patients arrive according to a Poisson process with a constant rate, and the service times has an exponential distribution. The arrivals are serviced on a FCFS (First-come-first-served) basis.

Figure 1 below depicted the M/S queuing model. The model architecture is composed of the waiting area, service node and departure components

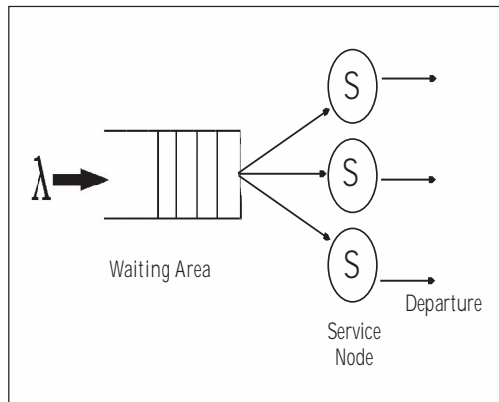


Fig 1: M/M/S queuing model

Performance metrics that succinctly depict the queuing model of the health-centre are given as follows:

The mean arrival rate

Let λ be the mean arrival rate and let n be the number of patients that entered the system. Also, let h be the number of observation hours. Then, the mean arrival rate is given by the formula,

$$\lambda = \frac{n}{h} \dots (3.1)$$

The mean service rate

At the history and the consulting rooms sections, patients were attended to one after the other. The service time for each patient was recorded when he or she was attended to.

Let s_1, s_2, \dots, s_n be the observed service time of patients. Let b be the start service time of patients and e be the finished service times of patients.

Then,

$$s_1 = e_1 - b_1$$

$$s_2 = e_2 - b_2$$

$$s_n = e_n - b_n$$

Hence, the mean service rate is given by;

$$\mu = \frac{1}{n} \sum_{i=1}^n s_i * \lambda_i \dots (3.2)$$

The mean waiting rate

For a system with a mean service rate μ, the average or mean service time is 1/ μ.

Hence,

$$W = W_q + \frac{1}{\mu} \dots (3.3)$$

where W_q is the average time in the waiting line and W is the mean waiting time in the system

$$P = \frac{\lambda}{s\mu} \dots (3.4)$$

where s is the number of servers and P gives an estimate of the utilization of the health centre i.e. the fraction of the system's service capacity that is being utilized in the average by arriving patients (λ).

$$L = \text{Average number of patient's waiting for service} = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{P^2}{1-P^2} \dots (3.5)$$

$$L_q = \text{Average time patients spent in the queue} = \frac{\lambda}{\mu-\lambda} \dots (3.6)$$

Probability that the system shall be idle is given by:

$$P_0 = \frac{1}{\left[\sum_{n=0}^{s-1} \frac{(\lambda/\mu)^n}{n!} + \frac{1}{s!} \left(\frac{\lambda}{\mu} \right)^s \frac{s\mu}{s\mu-\lambda} \right]} \dots (3.7)$$

The probability that there would be exactly n patients in the system:

$$P_n = \begin{cases} \frac{(\lambda/\mu)^n}{n!} P_0 & \text{if } 0 \leq n \leq s \\ \frac{(\lambda/\mu)^n}{s! s^{n-s}} P_0 & \text{if } n \geq s \end{cases} \dots (3.8)$$

RESULTS AND DISCUSSIONS

Calculations to predict the following performance measures analytically were carried using a Computer program written in Python language: the probability that there is no patient in system, the daily average number of patients in queue, the daily average number of patients in system, the daily average time a patient spends in waiting, the daily average time a patient spends in system and the probability that there is n number of patients in the health centre at a time (P_0, n, L, L_q, W_q, W and P_n respectively) from the data collected.

All of these are necessary for effective scheduling of hospital resources and optimum planning.

Simulation experiments were carried to obtain P_n assuming the values of n from 1 to 10 for each Week. Results for the concluding week is given in the table below:

Table 2: Estimated Probability values for n patients in the hospital

	Mon	Tues	Wed	Thur	Fri
n	P_n	P_n	P_n	P_n	P_n
0	0.86687	0.88901	0.76639	0.76588	0.62676
1	0.14286	0.11765	0.26600	0.26667	0.46660
2	0.01020	0.00692	0.03538	0.03556	0.10886
3	0.00049	0.00027	0.00314	0.00316	0.01693
4	1.74E-05	7.98E-06	0.00021	0.00021	0.00198
5	4.96E-07	1.88E-07	1.11E-05	1.12E-05	0.000184307
6	1.18E-08	3.68E-09	4.92E-07	4.99E-07	1.43E-05
7	2.41E-10	6.19E-11	1.87E-08	1.90E-08	9.55E-07
8	4.30E-12	9.10E-13	6.22E-10	6.34E-10	5.57E-08
9	6.83E-14	1.19E-14	1.84E-11	1.88E-11	2.89E-09
10	9.76E-16	1.40E-16	4.89E-13	5.01E-13	1.35E-10
	$\rho = 0.04761$	$\rho = 0.03921$	$\rho = 0.086666$	$\rho = 0.08888$	$\rho = 0.15553$

Further simulations were carried out to increase the number of servers from 2 to 3 to see the potential effect on performance delivery in the health centre. The results are given below:

Table 3: Performance values before Simulation to increase number of servers (S=2)

DAY	LQ	L	WQ	W
MON	3475.58065	3478.96065	342.7594329	343.0927663
TUE	4654.447215	4657.897215	449.705045	450.0383783
WED	1673621.678	1673625.345	152147.4253	152147.7586
THU	1083.995285	1087.795285	95.0873057	95.42063904
FRI	1169.684492	1173.501159	10.1558508	10.4891842

Table 4: Performance values after simulation to increase the number of servers (S=3)

DAY	LQ	L	WQ	W
MON	233.3238395	236.7571728	22.65279995	22.98613329
TUE	837.7320872	841.2320872	79.78400831	80.11734164
WED	804.814011	808.5473443	71.85839383	72.19172717
THU	1169.684492	1173.501159	56.1558508	60.4891842
FRID	80.23433757	83.38433757	8.490406093	8.823739426

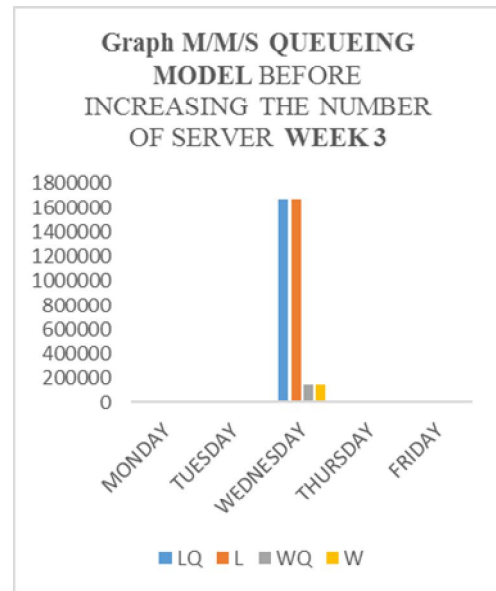


Fig 2: Graph before Simulation

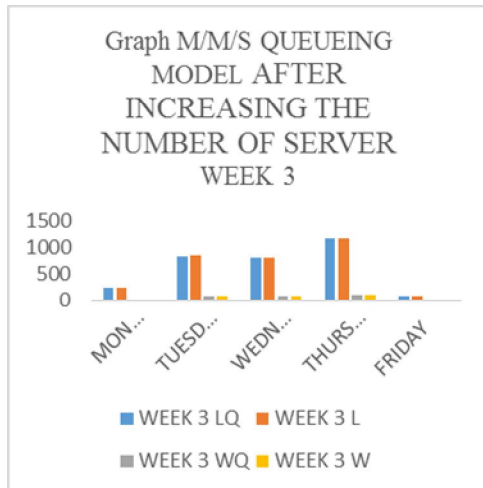


Fig 3: Graph after Simulation

Summary of Performance measure

From the table and graph the results show that on Monday week3 the server would be busy 4.79% of the time and idle 86.68% of the time. Also, the average number of patients in the waiting queue before and after were 3475.58065 and 233.3238395 and the average number of patients waiting in the system before and after were 3478.96065 and 236.7571728, the average time a patient spends in the queue before and after were 342.7594329 and 22.65279995 minutes and average time a patient spends in the system before and after were 343.0927663 and 22.98613329 minutes. For Tuesday the server would be busy 3.92% of the time and idle 88.909% of the time. Also, the average number of patients in the waiting queue before and after were 4654.447215 and 837.7320872 and the average number of patients waiting in the system before and after were 4657.897215 and 841.2320872, the average time a patient spends in the queue before and after were 449.705045 and 79.78400831 minutes and average a patient spends in the system before and after were 450.0383783 and 80.11734164minutes. For Wednesday the server would be busy 8.66% of the time and idle 76.639% of the time. Also, the average number of patients in the waiting queue before and after were 1673621.678 and 804.814011and the average number of patients waiting in the system before and after were 1673625.345 and 808.5473443, the average time a patient spends in the queue were 152147.4253 and 71.85839383minutes and average time a patient spends in the system before and after were 152147.7586 and 152147.7586minutes.

For Thursday the server would be busy 8.85% of the time and idle 76.588% of the time. Also, the average number of patients in the waiting queue before and after were 1083.995285 and 1169.684492 and the average number of patients waiting in the system before and after were 1087.795285 and 1173.501159, the average time a patient spends in the queue before and after

were 95.0873057 and 10.1558508 minutes and average a patient spends in the system were 95.42063904 and 60.4891842 minutes. And lastly for Friday the server would be busy 15.5% of the time and idle 62.67% of the time. Also, the average number of patients in the waiting queue before and after were 1169.684492 and 80.23433757and the average number of patients waiting in the system before and after is 1173.501159 and 83.38433757, the average time a patient spends in the queue before and after were 10.1558508 and 8.490406093minutes and average a patient spends in the system before and after were 10.4891842 and 8.82373942 6minutes.

Findings show that:

(i) Analysis of the system utilization factor of the servers at all the three weeks showed that among all the five number of days, the server of Friday week1 is the busiest of all with a utilization factor of 17.766%.

(ii) It was also observed that Friday week 1 had the highest mean arrival rate of 5.33 patients/hour. This is understandable as Fridays make the close of work for the week and patients may likely develop health issues that needed to be attended to at the health centre owing to the accumulated stress of work across the days of the week.

(iii)A reasonable chunk of the waiting time was expended at the reception (records) section where registration of patients takes place before access to the doctors.

(iii)Finally, It can be deduced that there was a considerably huge decrease in the waiting time and service time of patients when the server size was increased for each day of the Week.

It is suggested that the number of physicians should be increased and less time should be spent at the records section by putting measures in place for fast retrieval of patients' registration cards. A staffing plan should also be developed in the health centre in order to manage efficient shifting of personnel.

CONCLUSION

This work established queuing theory and modeling as an effective tool that can be used to make decision for optimal performance with regard to queuing challenges in health-care setting. The study determines the waiting, arrival and service times of patients at AAUA health-centre, to model a suitable queuing system and use simulation technique to validate the model to improve the quality of service delivery and patients' satisfaction. The study employed analytical and simulation methods to develop the suitable model. The result obtained from the simulation model the analysis of the entire system considered showed that queue length increases when the system is very

busy. The work therefore evaluated and predicted the system performance of AAUA Health-Centre in terms of service delivery and proposes solutions on needed resources to improve the quality of service offered satisfaction to the patients visiting the health centre.

Worthington D (2009) Reflections on queue modelling from the last 50 years. Journal of the Operations Research Society 60: s83–s92

REFERENCES

- Adedayo, A.O., Ojo, O. and Obamiro J.K. (2006) Operation Research in Decision Analysis and Production Management, Lagos: Pumark Nigeria Limited
- Agnihotri, S.R. and Taylor, P.F. (1991) Staffing a Centralized Appointment Scheduling Department in Lourdes Hospital
- Au-Yeung, S.W.M, Harder, U. and McCoy, E.J. (2009) 'Predicting patient arrivals to an accident and emergency department'. Journal of Emerging Medicine, 26:241-4
- Aziati Nor A.H. and Nur Salsabilah Binti Hamdan (2018). Application of Queuing Theory Model and Simulation to Patient Flow at the Outpatient Department
- Green V.L., Soares J., Giulio J. and Green, R. (2006) Using Queuing theory to increase the Effectiveness of Emergency Department Staffing, Academic Emergency Medicine, 13(1): 61-68
- Gupta D. (2013), Queuing for health care operation USA McGraw Hill Companies, Irwin Boston.
- Nosek, A.R., Wilson, P. J., (2001). Queuing Theory and Patients Satisfaction: A Review of Terminology, Trends and Applications to Pharmacy Practice'. Hospital Pharmacy, 36(3), pp. 275-279.
- Obamiro, J. K., (2006). Operations Research in Decision Analysis and Production Management. Pumark Nig. Ltd., Lagos.
- Obamiro, J.K., (2013). Effects of waiting line on patient satisfaction: Nigerian Hospital Experience. Journal of Economic Behaviour, 3(3):7-17
- Olorunsola S. A, Adeleke R. A and Ogunlade T. O (2014). Queueing Analysis of Patient Flow in Hospital, Ekiti state university, Ado Ekiti, Ekiti state, Nigeria.
- Peinus, W.R., Enyan A., Flanegen, P., Pim, B., Sallee, S. and Segrist J. (2000) A Proposed Scheduling Model to improve Use of Computed Tomography. Journal of Medical Systems, 24(1):69-71
- Singh, V., (2006) Use of Queuing Models in Health Care, Department of Health Policy and Management, University of Arkansas for medical science.
- Tucker J.B, Barone J.E, Cerere J., Blabey R.G. and Rha C.K. (1999). Using Queuing theory to determine operating room staffing needs. *The Journal of Trauma injury, infection and Critical care*, 46(1):10-16
- Vasanawala, S.S. and Desset T.S. (2005), Accommodation of Requests for Emergency US and CT: Application of Queuing Theory to Scheduling of Urgent Studies, Radiology, 238(1): 244-249