



On the Vibration of Prismatic Beam with Non Uniform Prestress under Exponential Loads Travelling at Varying Velocities

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Abstract

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In this paper, the analysis of prismatic beam with non uniform prestress subjected to both constant and exponential loads is investigated. The governing fourth order partial differential equation were analysed to determine the behavior of the system under consideration. The non-homogeneous governing equation is solved by means of Fourier Sine transform and Laplace transformation and convolution theory. The effects of some parameters such as non uniform prestress, elastic foundation and the length coordinates x of the beam are investigated through the numerical illustration of the obtained approximate analytical result are presented in plotted curves. The results show that the response amplitudes of the simply supported prismatic beam decrease as the foundation moduli K increases. Also, the displacements of simply supported prismatic beam resting on elastic foundation, for fixed value of K , decrease as the prestress N increases. Also the transverse deflections of the beam decrease as the length coordinates x decreases under the actions of moving concentrated loads.

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INTRODUCTION

For some decades, several investigations have been carried out by researchers on the problem of flexural vibrations of elastic structures (either beams or plates) to moving loads due to advances in transport and automobile engineering which results in high speed and heaviness of vehicle and other moving bodies (Fryba 1999; Adedowole 2008; Oni and Adedowole 2008).

Beam is a piece of horizontal or vertical structure that is capable of withstanding load primarily by resisting bending. The bending force is introduced into the material of the beam as a result of external load, weight, span and external reaction (Chang 2000). They are traditionally description of building or civil engineering structural elements, but smaller structures such as truck or automobiles frames, machine frames and other mechanical structural systems contains beam structures that are designed and analysed in a similar fashion. There are three basic types of beam which include:

- (i) Simple span, supported at both ends.
- (ii) Continuous, supported at more than two points.
- (iii) Cantilever, supported at one end with the other end overhanging and free.

It was recognized by the early researchers that the bending effect is the single most important factor in a transversely vibrating beam (Farghaly 1994; Gurgose 1996).

Furthermore, the problem of assessing the dynamic behaviour of structures carrying moving loads has been almost exclusively reserved in literature for moving loads moving at constant speeds (Oni 2004). The more practical cases when the velocities at which these loads move are no longer constants, but vary with time have received little attention in literature. This may be as a result of the complex space-time dependencies inherent in such problems. However, such practical problems as acceleration and braking of automobile on roadways and highway bridges, taking off and landing of air-crafts on runway and braking and acceleration forces in the calculation of rails and

railways bridges in which the motion is not uniform, but a function of time have intensified the need for study of the behaviour of structures under the action of loads moving with variable velocities.

LITERATURE REVIEW

Oni (1996) who considered the response of a non uniform thin beam resting on a constant elastic foundation to several moving masses. For the solution of the problem, he used the versatile technique of Galerkin to reduce the complex governing fourth order partial differential equation with variable and singular coefficients to a set of ordinary differential equations. The set of ordinary differential equations was later simplified and solved using modified asymptotic method of Struble.

Ogunbamike (2000) considered response of Timoshenko beams on Winkler foundation subjected to dynamic load by means of the Fourier

sine transformation. Shahin (2010) studied the response of a simply-supported beam on elastic foundation to repeated moving concentrated load by means of the Fourier sine transformation. The effects of some important parameters, such as the foundation stiffness, distance parameter and the travelling speed were studied. Sayad et al (2014) Studied vibration analyses of a tapered beam with exponentially varying thickness resting on Winkler foundation using the differential transform method. Usman et al (2015) examined the vibration of Timoshenko beam subjected to partially distributed moving load. The method of series solution and numerical method were used to solve the equation that governs the system. Adedowole (2016) worked on flexural motions under moving distributed masses of Beam-type structures on Vlasor foundation and having time dependent boundary conditions. He made use of Mindlin and Goodman's technique to transform the governing non-homogenous fourth order partial differential equations with non-homogenous boundary conditions into non-homogenous fourth order partial differential equation with homogenous boundary conditions.

Recently, Adedowole and Jimoh (2018) considered Dynamics analysis of a damped non uniform beam subjected to loads moving with variable velocity. they used a procedure involving the Galerkin's method and integral transform technique to solve the problem of a non-uniform beam when it is subjected to constant and harmonic variable magnitude moving loads. In particular, analytical solution in series form is obtained for the deflection of the elastic beam and the effects of foundation

stiffness K and the axial force N on the vibrating system were investigated.

Ogunyebi and Adedowole (2017) worked on the dynamic behavior of uniform Rayleigh beam with an accelerating distributed mass. The solution technique is based on Fourier Sine Integral transformation, the expansion of Heaviside function in series form and convolution theory. Jimoh *et al* (2018) studied the dynamic response of non-uniformly prestressed thick beam under distributed moving load travelling at varying velocity is investigated in this paper. They used a technique based on the method of Galerkin with the series representation of Heaviside function, was first used to transform the equation and thereafter the transformed equations were solved using Strubles asymptotic method and Laplace transformation techniques in conjunction with convolution theory. The displacement response for moving distributed force and moving distributed mass models for the dynamical problem are calculated for various time t and presented in plotted curves.

However, this work focuses on the vibration of prismatic beam with non uniform prestress under constant and exponential loads travelling at varying velocities. Numerical example involving a simply supported prismatic beam is studied and effects of some vital parameters are also presented.

MATHEMATICAL FORMULATION

The motion of a Bernoulli-Euler beam with non uniform prestress resting on an elastic foundation and under the action of constant and exponential loads moving with variable velocity is governed by the fourth order partial differential equation [10].

$$-D_x(x,t) + \overline{M}w_{tt}(x,t) - N(x)w_{xx}(x,t) + b_o w_t(x,t) + Kw(x,t) - P(x,t) = 0 \quad (1)$$

where x is the spatial co-ordinate $P(x,t)$ is the constant moving concentrated force acting on the beam, \overline{M} is the mass of the beam per unit length L , b_o is the material damping intensity, K is the uniform elastic foundation I_o is the moment of inertia of the beam $w(x,t)$ is the vertical response of the beam, $D_x(x,t)$ is the flexural moment and t is time.

The flexural moment acting on the beam across section is related to the vertical response as

$$D_x(x, t) = -EI(x)w_{xx}(x, t) \quad (2)$$

where $EI(x)$ the flexural rigidity of the beam, E is the young modulus

In this paper, the non uniform prestress $N(x)$ is given as

$$N(x) = N_o \left(1 + \frac{\sin \pi x}{L} \right) \quad (3)$$

N_o is the beam characteristics at $x = 0$.

The Boundary Conditions

The boundary conditions depend on the constraints at the beam ends, however for a simply supported beam whose length is L , the vertical displacement at the beam ends are given as:

$$w(0, t) = w(L, t) = 0 \quad (5a)$$

$$w_{xx}(0, t) = w_{xx}(L, t) = 0 \quad (5b)$$

It is assumed that the initial conditions are

$$w(x, 0) = 0 = w_t(x, 0) \quad (6)$$

Case I. Dynamic behavior of prismatic beam to constant magnitude loads.

The constant vertical excitation acting on the beam is chosen as

$$Q(x, t) = P\delta(x - f(t)) \quad (7)$$

The concentrated load is assumed to be of mass M and the time t is assumed to be limited to that interval of time within the mass on the beam, that is;

$$0 \leq f(t) \leq L \quad (9)$$

The body moves with non uniform velocity such that the motion of the contact of the moving load is given by

$$X_p = f(t) \quad (10)$$

The distance covered by the load on the same structure at any given instance of time t is given as

$$f(t) = x_o + x_1 \quad (11)$$

where x_o is the equilibrium position of the longitudinal oscillating load, x_1 is the distance covered from equilibrium position x_o .

From equations of motion we have

$$x_1 = \frac{(c + v)t}{2} \quad (12a)$$

$$v = c + at \quad (12b)$$

where c is initial speed, v is the final speed and a is the constant acceleration

Substituting equations (7) and (11) into equation (1), one obtains

$$\begin{aligned} &EI_o w_{xxxx}(x, t) + \bar{M}w_{tt}(x, t) \\ &- N_o \left(1 + \frac{\sin \pi x}{L} \right) w_{xx}(x, t) + b_o w_t(x, t) \\ &+ Kw(x, t) = P\delta \left[x - \left(x_o + \frac{(c + v)t}{2} \right) \right] \end{aligned} \quad (13)$$

Method of Solution

Equation (13) is a fourth order partial differential equation, which in addition to being singular has variable coefficients. Firstly, by virtue of the boundary conditions the fourth order equation will be reduced to second order equation by applying the finite Fourier Sine integral transform, with respect to x .

The integral transform is defined

$$u(m, t) = \int_0^L w(x, t) \sin \frac{m\pi x}{L} dx \quad (14)$$

With the inverse

$$w(x, t) = \frac{2}{L} \sum_{m=1}^{\infty} u(m, t) \sin \frac{m\pi x}{L} \quad (15)$$

Thus applying (14) to the (10), one obtains

$$\sum_{i=1}^n \left[\left\{ \frac{EI_o}{4} \left(\frac{i\pi}{L} \right)^4 + N_o \left(1 + \frac{\sin \pi x}{L} \right) \left(\frac{i\pi}{L} \right)^2 + K \right\} u(m,t) + \overline{M} \ddot{u}(m,t) + b_o \dot{u}(m,t) \right] \sin \frac{i\pi x}{L} = P \delta \left[x - \left(x_o + \frac{(c+v)t}{2} \right) \right] \quad (16)$$

By orthogonally condition, equation (16) becomes

$$\frac{EI_o}{4} \left(\frac{m\pi}{L} \right)^4 I_1 u(m,t) + N_o \left(\frac{m\pi}{L} \right)^2 \times (I_1 + I_2) u(m,t) + \overline{M} I_1 \ddot{u}(m,t) + b_o I_1 \dot{u}(m,t) + K I_1 u(m,t) = \int_0^L P \delta \left[x - \left(x_o + \frac{(c+v)t}{2} \right) \right] \sin \frac{n\pi x}{L} dx \quad (17)$$

Where

$$I_1 = \int_0^L \sin \frac{m\pi x}{L} \sin \frac{n\pi x}{L} dx, \quad I_2 = \int_0^L \sin \frac{\pi x}{L} \sin \frac{m\pi x}{L} \sin \frac{n\pi x}{L} dx \quad (18)$$

Further rearrangements and simplifications of equation (17) we obtain

$$\sum_{m=1}^{\infty} [\beta_1(m,n) \ddot{u}_m(t) + \beta_2(m,n) \dot{u}_m(t) + \beta_3(m,n) u_m(t)] = P \sin \frac{n\pi}{L} (x_o + v_o t) \quad (19)$$

where

$$v_o = \frac{(c+v)}{2}, \quad \beta_1(m,n) = \overline{M} \frac{L}{2},$$

$$\beta_2(m,n) = b_o \frac{L}{2},$$

$$\beta_3(m,n) = \left(\frac{EI_o}{4} \left(\frac{m\pi}{L} \right)^4 + K \right) I_1 + N_o \left(\frac{m\pi}{L} \right)^2 (I_1 + I_2) \quad (20a)$$

Equation (19) is the second order ordinary differential equation with constant coefficient to a transformation

In what follow we subject the system of ordinary differential equation (19) to a Laplace transform defined as

$$(\sim) = \int_0^{\infty} (\cdot) e^{-st} dt \quad (20b)$$

In conjunction with the initial conditions defined in (6), yields the following algebraic equation

$$[\beta_1(m,n)s^2 + \beta_2(m,n)s + \beta_3(m,n)] u_m(s) = P \left[\frac{a_o S}{S^2 + \theta^2} + \frac{b_o \theta}{S^2 + \theta^2} \right] \quad (21)$$

where

$$a_o = \left(\sin \frac{n\pi x_o}{L} \right), \quad b_o = \left(\cos \frac{n\pi x_o}{L} \right)$$

$$\text{and } \theta = \left(\frac{n\pi v_o t}{L} \right) \quad (22)$$

Subjecting equation (21) for further simplification yields

$$y_i(s) = P_k \left[\left(\frac{s}{s^2 + \theta^2} \cdot \frac{a_o}{S - d_1} + \frac{\theta}{s^2 + \theta^2} \cdot \frac{b_o}{S - d_1} \right) * \left(\frac{s}{s^2 + \theta^2} \cdot \frac{a_o}{S - d_1} + \frac{\theta}{s^2 + \theta^2} \cdot \frac{b_o}{S - d_1} \right) \right] \quad (23)$$

where

$$P_k = \frac{P_o}{(d_1 - d_2)}, d_1 = \frac{-\beta_2 + \sqrt{\beta_2^2 - 4\beta_1\beta_3}}{2\beta_1}$$

and $d_2 = \frac{-\beta_2 - \sqrt{\beta_2^2 - 4\beta_1\beta_3}}{2\beta_1}$ (24)

To obtain the Laplace inversion of equation (21), we shall adopt the following representations

$$g_1(s) = \frac{s}{s^2 + \theta^2}, g_2(s) = \frac{\theta}{s^2 + \theta^2}$$

and $f_2(s) = \frac{1}{s - d_2}$ (25)

So that the Laplace inversion of each term of the RHS (23) is the convolution of f_i s and g defined by as

$$f_s * g = \int_0^t f_i(t-u)g(u)du, \quad i = 1,2$$
 (26)

Thus the Laplace inversion of (23) is given by

$$u_m(t) = P_k \left\{ \left(\frac{e^{d_1 t}}{d_1} [a_0 I_3 + b_0 I_4] \right) - \left(\frac{e^{d_2 t}}{d_2} [a_0 I_5 + b_0 I_6] \right) \right\}$$
 (27)

where

$$I_3 = \int_0^t e^{-d_1 u} \cos \theta \tau d\tau$$

$$I_4 = \int_0^t e^{-d_1 u} \sin \theta \tau d\tau$$

$$I_5 = \int_0^t e^{-d_2 u} \cos \theta \tau d\tau$$

$$I_6 = \int_0^t e^{-d_2 u} \sin \theta \tau d\tau$$
 (28)

Evaluating integrals in equation (28) we have

$$I_3 = \frac{-\theta e^{d_1 t} \sin \theta t + d_1 - d_1 e^{d_1 t} \cos \theta t}{(\theta^2 + d_1^2)}$$

$$I_4 = \frac{-\theta e^{d_1 t} \cos \theta t + d_1 - d_1 e^{d_1 t} \sin \theta t}{(\theta^2 + d_1^2)}$$

$$I_5 = \frac{-\theta e^{d_2 t} \sin \theta t + d_2 - d_2 e^{d_2 t} \cos \theta t}{(\theta^2 + d_2^2)}$$

$$I_6 = \frac{-\theta e^{d_2 t} \cos \theta t + d_2 - d_2 e^{d_2 t} \sin \theta t}{(\theta^2 + d_2^2)}$$
 (29)

Further simplification of equation (27) yields

$$u_m(t) = P_k \left[\left(\frac{e^{d_1 t}}{d_1(\theta^2 + d_1^2)} \right) \times [a_0(d_1 - \theta e^{-d_1 t} \sin \theta t - d_1 e^{-d_1 t} \cos \theta t) + b_0(\theta - \theta e^{-d_1 t} \cos \theta t - d_2 e^{-d_2 t} \sin \theta t)] \right]$$

$$\left\{ \frac{e^{d_2 t}}{d_2(\theta^2 + d_2^2)} \right\} \times [a_0(d_2 - \theta e^{-d_2 t} \sin \theta t - d_2 e^{-d_2 t} \cos \theta t) + b_0(\theta - \theta e^{-d_1 t} \cos \theta t - d_1 e^{-d_1 t} \sin \theta t)]$$
 (30)

Substituting equation (30) into equation (15) which on inversion yields

$$w(x,t) = \sum_{m=1}^n P_k \left[\left(\frac{e^{d_1 t}}{d_1(\theta^2 + d_1^2)} \right) \times [a_0(d_1 - \theta e^{-d_1 t} \sin \theta t - d_1 e^{-d_1 t} \cos \theta t) + b_0(\theta - \theta e^{-d_2 t} \cos \theta t - d_2 e^{-d_2 t} \sin \theta t)] \right]$$

$$- \left\{ \frac{e^{d_2 t}}{d_2(\theta^2 + d_2^2)} \right\} \times [a_0(d_2 - \theta e^{-d_2 t} \sin \theta t - d_2 e^{-d_2 t} \cos \theta t) + b_0(\theta - \theta e^{-d_1 t} \cos \theta t - d_1 e^{-d_1 t} \sin \theta t)] \Big] \sin \frac{m\pi x}{L}$$
 (31)

Equation (31) is the transverse deflection of the uniform Bernoulli beam with non uniform prestress

under the action of concentrated forces travelling with variable velocity

Case II Dynamic Behavior of Prismatic Beam to Exponential Variable Magnitude Loads.

The dynamic behavior of prismatic beam when subjected to exponential variable magnitude moving load is investigated in this section. Thus, the load $P(x,t)$ is given as

$$Q(x,t) = Pe^{\Omega t} \delta \left[x - \left(x_0 + \frac{(c+v)t}{2} \right) \right] \quad (32)$$

where Ω is the circular frequency of the exponential force and all parameters are as defined previously substituting equation (32) in equation (1), vibration of the beam is then described by the equation

$$\begin{aligned} &EI_o w_{xxxx}(x,t) + \bar{M}w_{tt}(x,t) \\ &- N_o \left(1 + \frac{\sin \pi x}{L} \right) w_{xx}(x,t) \\ &+ b_o w_t(x,t) + Kw(x,t) \\ &= Pe^{\Omega t} \delta \left[x - \left(x_0 + \frac{(c+v)t}{2} \right) \right] \end{aligned} \quad (33)$$

Equation (33) is the governing equation describing the motion of prismatic beams subjected to moving loads with variable velocity of varying magnitude. Like in the previous section, as closed-form solution to equation (33) is sought.

To this effect, use is made of an assumed mode method, finite Fourier Sine integral transform, and by this method the transverse deflection $w_b(x,t)$ of prismatic beam under the action of variable magnitude moving load can be written as

$$u_b(r,t) = \int_0^L w_b(x,t) \sin \frac{r\pi x}{L} dx \quad (34)$$

With the inverse

$$w_b(x,t) = \frac{2}{L} \sum_{m=1}^{\infty} u_b(r,t) \sin \frac{r\pi x}{L} \quad (34b)$$

In view of equation (34) and following the same arguments as in the previous section and after some simplifications and rearrangements equation (33) becomes

$$\begin{aligned} &\sum_{m=1}^{\infty} [\beta_1(m,n)\ddot{u}_m(t) + \beta_2(m,n)\dot{u}_m(t) + \beta_3(m,n)u_m(t)] \\ &= Pe^{\Omega t} \sin \frac{n\pi}{L} (x_0 + v_0 t) \end{aligned} \quad (35)$$

where v_r is the velocity of the r^{th} particle of the system and other parameter are as defined previously

Without loss of generality, considering only the r th particle of the dynamical system yields

$$\begin{aligned} &\beta_1(r,n)\ddot{u}_r(t) + \beta_2(r,n)\dot{u}_r(t) + \beta_3(r,n)u_r(t) \\ &= Pe^{\Omega t} \sin \frac{n\pi}{L} (x_0 + v_0 t) \end{aligned} \quad (36)$$

Equation (36) is analogous to equation (19), thus subjecting equation (36) to Laplace transform in conjunction with the boundary conditions stated (5) and using convolution theory we obtain

$$\begin{aligned} u_r(t) = P_k &\left\{ \frac{e^{\eta_3 t}}{d_1(\eta_1^2 + \theta^2)} \right. \\ &\times (\eta_1(b_0 \sin \theta t + a_0 \cos \theta t) \\ &\left. + \theta(a_0 \sin \theta t - b_0 \cos \theta t)) \right\} \\ &- \left\{ \frac{e^{\eta_4 t}}{d_2(\eta_2^2 + \theta^2)} \right. \\ &\times [\eta_2(b_0 \sin \theta t + a_0 \cos \theta t) \\ &\left. + \theta(a_0 \sin \theta t - b_0 \cos \theta t)] \right\} \end{aligned} \quad (37)$$

Where

$$\begin{aligned} \eta_1 &= \Omega - d_1, & \eta_2 &= \Omega - d_2, \\ \eta_3 &= \eta_1 + d_1, & \eta_4 &= \eta_2 + d_2 \end{aligned} \quad (38)$$

which on inversion yields

$$\begin{aligned} w_r(x,t) &= \frac{2}{L} \sum_{r=1}^{\infty} P_k \left\{ \frac{e^{\eta_3 t}}{d_1(\eta_1^2 + \theta^2)} \right. \\ &\times (\eta_1(b_0 \sin \theta t + a_0 \cos \theta t) \end{aligned}$$

$$\begin{aligned}
 & + \theta(a_0 \sin \theta t - b_0 \cos \theta t) \Big\} \\
 & - \left\{ \frac{e^{\eta_2 t}}{d_2(\eta_2^2 + \theta^2)} \right. \\
 & \times [\eta_2(b_0 \sin \theta t + a_0 \cos \theta t) \\
 & \left. + \theta(a_0 \sin \theta t - b_0 \cos \theta t) \right\} \sin \frac{r\pi x}{L} \\
 & \qquad \qquad \qquad (39)
 \end{aligned}$$

Equation (39) represents the transverse displacement response of prismatic beam under actions of exponential moving distributed loads.

DISCUSSION ON THE CLOSED FORM SOLUTION

The transverse displacement of an elastic beam may increase without bound. Thus one is interested in resonance conditions. Equation (31) clearly depicts that the non prismatic beam resting on exponentially decaying foundation traverse by a constant moving load will grow without bound whenever

$$d_1 = d_2, d_1^2 = -\theta^2 \text{ or } d_2^2 = -\theta^2 \quad (40)$$

While equation (39) shows that the same beam under the action of harmonic variable magnitude moving loads will experience resonance effects whenever

$$d_1 = d_2, \theta^2 = -\eta_1^2 \text{ or } \theta^2 = -\eta_2^2 \quad (42)$$

Remarks on Numerical Results

This section investigates the dependence of the prismatic beam response on elastic foundation K, non uniform prestress N and, load position X. Making use of the theoretical analysis described in the previous sections and in order to carry out the calculations of practical interest in dynamics of structure and engineering designs, a beam of length 12.92m is considered for illustrative examples. It is assumed that the mass travels at varying velocity with initial velocity of 8.123m/s, the modulus of elasticity $E = 2.109 \times 10^9$, the moment of inertia $I = 2.37698 \times 10^{-3}$ and $\Omega = 0.2$

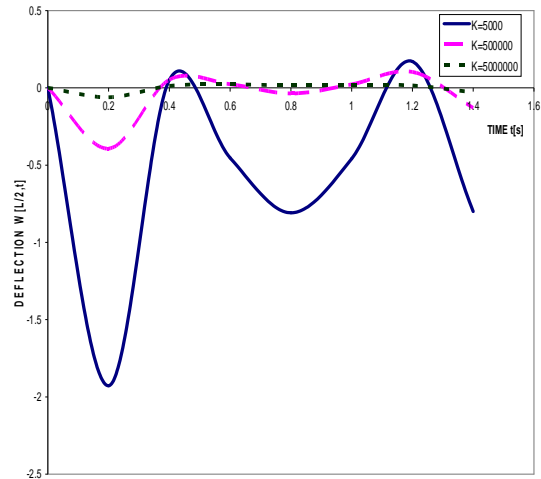


Fig. 1: Displacement response of prismatic beam subjected to constant magnitude moving loads for various values of foundation modulus K and for fixed values of X and N

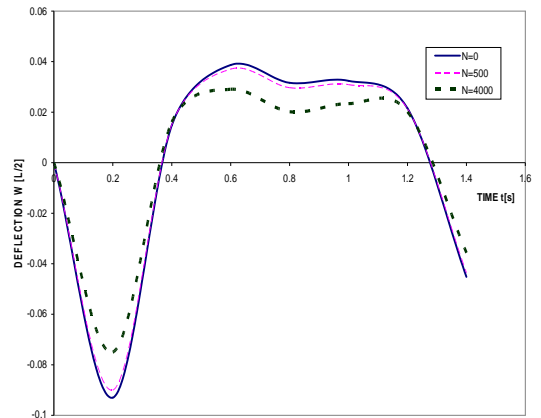


Fig. 2: Transverse displacement response of prismatic beam subjected to constant magnitude moving loads for various values of axial force N and for fixed value of K and X

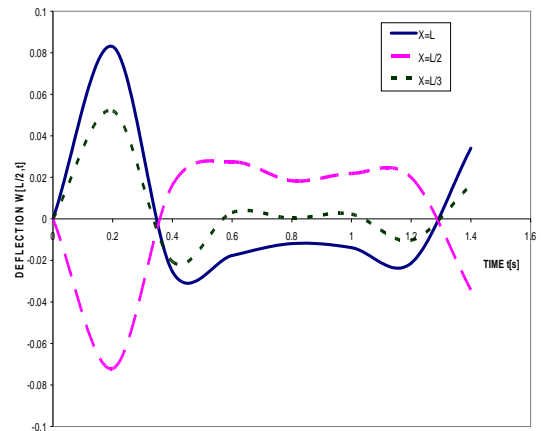


Fig. 3: Transverse displacement of a beam under the actions of exponential loads travelling at variable velocity for various values of load positions x and for fixed values of K and N

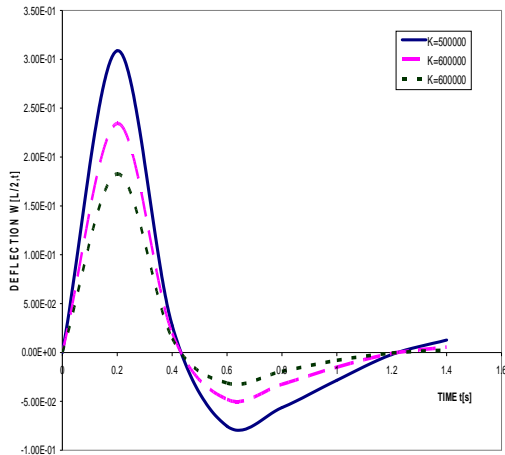


Fig. 4: Displacement response of prismatic beam subjected to exponential moving loads for various values of foundation modulus K and for fixed values of X and N

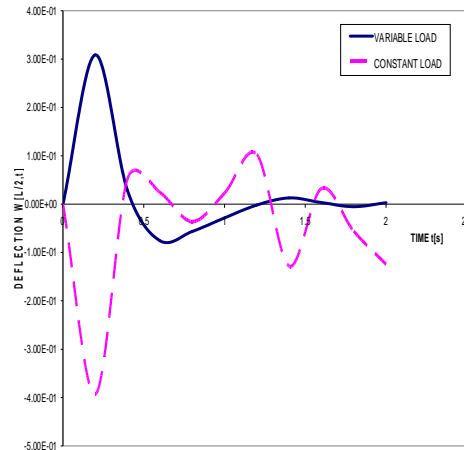


Fig 7: Comparison of the displacement response of constant and exponential loads cases of a prismatic beam for fixed values of X , K and N

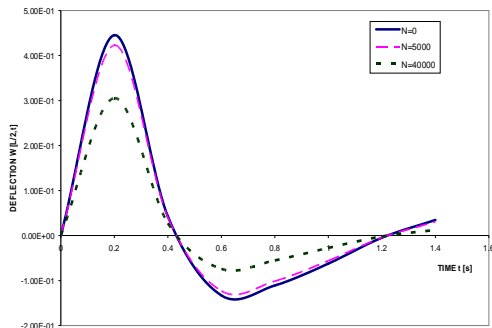


Fig 5: Transverse displacement response of prismatic beam subjected to exponential moving loads for various values of axial force N and for fixed value of K and X

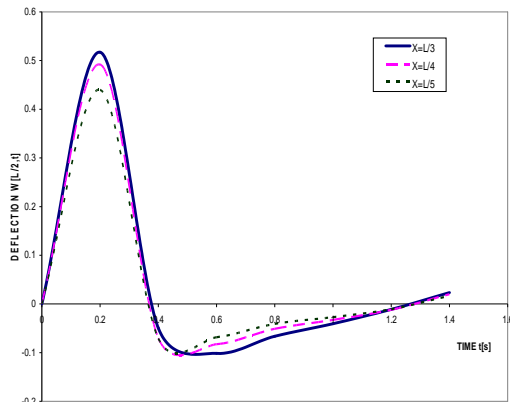


Fig. 6: Transverse displacement of a beam under the actions of exponential loads travelling at variable velocity for various values of load positions x and for fixed values of K and N

Figures 2 and 5 show the influence of prestress N on the deflection profile of simply supported prismatic beam in both cases of constant and exponential moving loads respectively. The graphs show that higher values of prestress N decrease the vibration of the beams. For various time, t the displacement of the beam for fixed K , N and for various values of spartial coordinates X are shown in figures 3 and 6. It is deduced from these figures that the interval at which structure is supported affect the response amplitude of the beam significantly.

Finally, figure 7 depicts the comparison of the traversed displacement of constant and exponential moving loads for fixed values foundation modulus K , prestress N and spartial coordinates X . Clearly, the response amplitude of constant magnitude moving load is higher than that of the exponential magnitude moving load.

CONCLUSION

In this investigation, transverse vibration of prismatic beam with non uniform prestress and under constant and exponential loads travelling at varying velocity is presented. The solution technique is based on Fourier Sine Integral transformation and convolution theory.

Numerical analysis is also carried out and results show the following:

- a. for the constant and exponential magnitude moving loads problems. The response amplitudes of the beam decrease with an increase in the foundation constant K .

b. as the value of prestress N increases, the displacement response of a prismatic beam decreases for fixed values of foundation modulus, K and spartial coordinates X .

c. the responses amplitude of the variable harmonic load is smaller than that of the constant magnitude load. This shows that resonance is reached earlier in constant magnitude problem.

Further research can be carried out on this work when the beam is subjected to repeated loads

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