



Dynamical Analysis of Non Prismatic Rayleigh Beam Resting on Non-Winkler Foundation and Subjected to Traveling Distributed Forces

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Abstract

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The dynamic response of a simply supported non prismatic Rayleigh beam under distributed moving forces resting on Winkler elastic foundation with exponential stiffness is investigated. For the solution of the fourth order partial differential equation with singular and variable coefficients, use is made of the technique based on the Galerkin's method, integral transformation and convolution theory. The numerical results are presented in plotted curves. The results show that the response amplitudes of the simply supported non-prismatic Rayleigh beam decrease as the foundation moduli F_s increase. Also, the displacements of simply supported non-prismatic Rayleigh beam resting on Winkler elastic foundation, for fixed value of F_s , decrease as the prestress N increases. Also the transverse deflections of the beam decrease as the length coordinate x decreases under the actions of moving distributed loads. The results show that the critical speed for the moving constant load case is reached earlier than that for the harmonic variable moving load.

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INTRODUCTION

The study of dynamical behaviour of structures such as beams and plates, under the action of moving loads has attracted the attention of several researchers in Engineering, Applied Physics and Applied Mathematics. Notable among such researchers are Frybal (1972), Kolousek (1967), Gbadeyan et al (1995) and Stanisic et al (1974).

In a problem of beam under moving load like this, the effect of the mass of the load is of great importance since the position of the load changes continuously. Extensive work has been done on this class of dynamical problems when the structural members have uniform cross-sections. Work on practical problems involving non-uniform Bernoulli-Euler beam which is under the action of moving concentrated load is very rare in literature. Such one dimensional structures are of variable cross-sections and as such their properties vary with respect to the spatial coordinates along the span of the structure. Worthy of mention in this area is the work of Kolousek (1967).

LITERATURE REVIEW

Several researchers have made tremendous efforts in the study of dynamics of structures under moving loads; these include Gbadeyan and Oni (1995), Oni and Adedowole (2008), Oni and Awodola (2008) and Ogunyebi (2017). Meera Saheb *et al.* (2007) conducted a free vibration analysis for uniform Timoshenko beams using the coupled displacement field method. Shahin (2010) studied the response of a simply-supported beam on elastic foundation to repeated moving concentrated load by means of the Fourier sine transformation. The effects of some important parameters, such as the foundation stiffness, distance parameter and the travelling speed were studied. In all of these, considerations have been limited to cases of uniform beams.

Among the works is also the work of Oni (1996) who considered the response of a non-uniform thin beam resting on a constant elastic foundation to several moving masses. For the solution of the

problem, he used the versatile technique of Galerkin to reduce the complex governing fourth order partial differential equation with variable and singular coefficients to a set of ordinary differential equations. Mahmoud *et al* (2013) applied the differential transformation method (DTM) for free vibration analysis of beams with uniform and non-uniform cross sections. Ojih *et al* (2013) investigated the dynamic response of non-uniform Rayleigh beam resting on Pasternak foundation and subjected to concentrated loads travelling at constant velocity.

Furthermore, the problem of assessing the dynamic behaviour of structures carrying moving loads has been almost exclusively reserved in literature for moving loads have been idealized as moving concentrated loads which act at a certain point on the structure and along single line in space. That is, the moving load is modeled as a lumped load. In practice, loads are actually distributed over a small segment or over the entire length of the structural member as they transverse the structure. Such moving loads are termed uniformly distributed loads. Concentrated forces are mere mathematical idealization, but cannot be found in the real world, where all forces are body forces acting over an area. For this reason, Dada (2002) worked on the vibration analysis of elastic plates under uniform partially distributed loads and Adetunde (2003) studied the dynamic response of Rayleigh beam carrying an added mass and traversed by uniform partially distributed moving loads. Omolofe *et al* (2011) studied exact vibration solution for initially stressed beams on elastic foundation and subjected to partially distributed masses. The problem of the flexural vibrations of structural members with general boundary conditions and resting on Winkler foundation was taken up by Andi (2013). The governing equation of motion was solved when the moving loads were modelled as distributed moving loads. Very recently, Ogunbanke (2014) considered dynamics of elastic structures resting on bi-parametric Vlasov foundation and under distributed masses travelling at varying velocities.

This present work focuses on the analytical solution of a non prismatic Rayleigh beam resting on non-Winkler foundation with exponential stiffness subjected to moving distributed forces. The analysis shows the dynamic effect of foundation constant F_s , rotatory inertia correction factor R_o and prestress N on the structure as presented in the plots.

Definition of the Problem

Consider a non prismatic simply supported beam with length L under distributed load. The distributed loads M_i move across the beam

starting at time $t = 0$ with constant velocity v_i . The equation of motion for the system is given by the fourth order partial differential equation Frybal (1972)

$$\begin{aligned} &\mu(x)z_{tt}(x,t) + b(x)z_t(x,t) + F(x)z(x,t) \\ &- q_x(x,t) - \mu(x)R_o z_{xxt}(x,t) - (N + G)z_{xx}(x,t) \\ &= P(x,t) \end{aligned} \tag{1}$$

$$q(x,t) = D_x(x,t) \tag{2}$$

where $q(x,t)$ is the shear force, $P(x,t)$ is the constant moving distributed force acting on the beam, μ is the mass of the beam per unit length L , b is the material damping intensity, $z(x,t)$ is the vertical response of the beam, $D_x(x,t)$ is the flexural moment and t is time, G is the shear modulus and R_o is the rotatory inertia.

The flexural moment acting on the beam across section is related to the vertical response as

$$D_x(x,t) = -EI(x)z_{xx}(x,t) \tag{3}$$

where $EI(x)$ the flexural rigidity of the beam, E is the young modulus

The distribution of the non prismatic characteristics may be assumed as power functions. The parameters γ and n , are used to approximate the actual non uniformity of the beam given as

$$\begin{aligned} I(x) &= I_o(1 + \gamma x)^{n+2}, \\ \mu(x) &= \mu_o(1 + \gamma x)^n, \\ b(x) &= b_o(1 + \gamma x)^n \end{aligned} \tag{4}$$

where $I(x)$ is the variable moment of inertia of the beam, I_o , μ_o and b_o are the beam characteristics at $x = 0$.

The boundary conditions depend on the constraints at the beam ends, however for a simply supported beam whose length is L , the vertical displacement at the beam ends are given as

$$\begin{aligned} z(0,t) &= z(L,t) = 0, \\ z_{xx}(0,t) &= z_{xx}(L,t) = 0. \end{aligned} \tag{5}$$

It is assumed that the initial conditions are

$$z(x,0) = 0 = z_{,tt}(x,0). \tag{6}$$

Case 1:

Analysis of non-prismatic rayleigh beam with exponentially decaying foundation to constant magnitude moving distributed forces.

In this paper, we adopt Omolofe et al (2011) and define the exponential decaying elastic foundation $F(x)$ as

$$F(x) = F_s e^{-\lambda x} \tag{7}$$

where λ is a constant and F_s is the elastic foundation constant.

Furthermore, the constant vertical excitation acting on the beam is chosen as

$$P(x,t) = PH(x - v_i t). \tag{8}$$

The distributed load is assumed to be of mass M and the time t is assumed to be limited to that interval of time within the mass on the beam, that is

$$0 \leq f(t) \leq L \tag{9}$$

and $H(x - vt)$ is the Heaviside function defined as

$$H(x - vt) = \begin{cases} 1, & x > 0 \\ 0, & x < 0 \end{cases} \tag{10a}$$

with the properties

$$\frac{d}{dx} \{H(x - vt)\} = \delta(x - vt) \tag{10b}$$

where $\delta(x - vt)$ represents the Dirac delta function and $H(x - vt)$ is a typical engineering function made to measure engineering applications which often involved functions that are either “off” or “on”. v_i is the velocity of the i^{th} particle of the system, t is the travelling time. Substituting equations (2), (3), (4) and (7) into equation (1) and taking $n = 1$ for simplicity yields

$$\begin{aligned} EI_o(x) & \left[\frac{\partial^2}{\partial x^2} \left((1 + \gamma x)^3 z_{,xx}(x,t) \right) \right] \\ & + \mu_o(1 + \gamma x) z_{,tt}(x,t) - \mu_o(1 + \gamma x) R_o z_{,xxt}(x,t) \\ & - Nz_{,xx}(x,t) - Gz_{,xx}(x,t) + b_0(1 + \gamma x) z_{,t}(x,t) \\ & + F_s e^{-\lambda x} z(x,t) = PH(x - v_i t). \end{aligned} \tag{11}$$

To the authors best of knowledge, a closed form solution to the fourth order partial differential equation (11) governing the motion of the beam under the actions of moving force, does not exist. Existing method of solutions are Meera Saheb et al (2007) displacement field method, Shahin (2010) Fourier sine transformation and Mahmoud et al (2013) differential transformation method (DTM). In this study, a solution technique called Galerkin’s method would be used. It is desirable to obtain some vital information about the vibrating system.

Approximate Analytical Solution

To solve the beam problem equation, we shall use the versatile solution technique called Galerkin’s method often used in solving diverse problems involving mechanical vibrations Oni (1996). The equation of the motion of an element of the beam is generally symbolically written in the form

$$\Gamma z(x,t) - P(x,t) = 0 \tag{12}$$

where Γ is the differential operator with variable coefficients. The solutions of the equation (11) is expressed as

$$z(x,t) = \sum_{i=1}^n y_i(t) Q_i(x) \tag{13}$$

where $y_i(t)$ are coordinates in modal space and $Q_i(x)$ are the normal modes of free vibration written as

$$\begin{aligned} Q_i(x) & = \sin \theta_i x + A_i \cos \theta_i x + B_i \sinh \theta_i x \\ & + C_i \cosh \theta_i x. \end{aligned} \tag{14}$$

Thus, for a simply supported beam, it can be shown that

$$A_i = B_i = C_i = 0 \text{ and } \theta_i = \frac{i\pi}{L}. \tag{15}$$

Thus in view of equation (15), equation (13) can be written as

$$z(x, t) = \sum_{i=1}^n y_i(t) \sin \frac{i\pi x}{L} \quad (16)$$

Substituting equation (16) into the governing equation (11) and after some simplifications and arrangements one obtains

$$\begin{aligned} & EI_o(x) \left[\frac{\partial^2}{\partial x^2} \left((1 + 3\gamma x + 3\gamma^2 x^2 + \gamma^3 x^3) \right. \right. \\ & \times \left. \frac{\partial^4}{\partial x^4} \sum_{i=1}^n y_i(t) \sin \frac{i\pi x}{L} \right) \\ & + (3\gamma + 6\gamma^2 x + 3\gamma^3 x^2) \\ & \times \frac{\partial^3}{\partial x^3} \sum_{i=1}^n y_i(t) \sin \frac{i\pi x}{L} \\ & + (6\gamma^2 + 6\gamma^3 x) \frac{\partial^2}{\partial x^2} \sum_{i=1}^n y_i(t) \sin \frac{i\pi x}{L} + \\ & - R_o \mu_o (1 + \gamma x) \frac{\partial^4}{\partial x^2 \partial t^2} \sum_{i=1}^n y_i(t) \sin \frac{i\pi x}{L} \\ & - N \frac{\partial^2}{\partial x^2} \sum_{i=1}^n y_i(t) \sin \frac{i\pi x}{L} \\ & - G \frac{\partial^2}{\partial x^2} \sum_{i=1}^n y_i(t) \sin \frac{i\pi x}{L} \\ & + (1 + \gamma x) \frac{\partial}{\partial x} \sum_{i=1}^n y_i(t) \sin \frac{i\pi x}{L} \left. \right] \\ & + \mu_o (1 + \gamma x) \frac{\partial^2}{\partial t^2} \sum_{i=1}^n y_i(t) \sin \frac{i\pi x}{L} \\ & + b_o (1 + \gamma x) \frac{\partial}{\partial t} \sum_{i=1}^n y_i(t) \sin \frac{i\pi x}{L} \\ & + F_s e^{-\lambda x} \sum_{i=1}^n y_i(t) \sin \frac{i\pi x}{L} = PH(x - v_i t) \quad (17) \end{aligned}$$

To simplify equation (17) further, we use orthogonality condition. Thus, multiplying equation (17) by $\sin \frac{j\pi x}{L}$ and integrating from 0 to L with respect to x after some simplifications and rearrangements yield

$$\begin{aligned} & b_1(i, j) \ddot{y}_i(t) + b_2(i, j) \dot{y}_i(t) + b_3(i, j) y_i(t) \\ & = -P \left(\frac{j\pi}{L} \right) \cos \frac{j\pi v t}{L} \quad (18a) \end{aligned}$$

where

$$\begin{aligned} & b_1(i, j) = \mu_o [I_1 + \gamma I_2] \\ & b_2(i, j) = b_o \frac{I_1}{2} \\ & b_3(i, j) = EI_o [a_1 - a_2 - a_3] + a_4 \\ & a_1 = \left(\frac{i\pi}{L} \right)^4 [I_1 + 3\gamma I_2 + 3\gamma^2 I_3 + \gamma^3 I_4] \\ & a_2 = \left(\frac{i\pi}{L} \right)^3 [3\gamma I_5 + 6\gamma^2 I_6 + 3\gamma^3 I_7] \\ & a_3 = \left(\frac{i\pi}{L} \right)^2 [6\gamma^2 I_1 + 6\gamma^3 I_2] \quad (18b) \end{aligned}$$

In what follow we subject the second order ordinary differential equation (18a) to a Laplace transform defined as

$$(\sim) = \int_0^\infty (\cdot) e^{-st} dt \quad (18c)$$

In conjunction with the initial conditions defined in (6), yields the following algebraic equation

$$\begin{aligned} & (b_1(i, j)s^2 + b_2(i, j)s + b_3(i, j)) y_i(s) \\ & = P_o \frac{s}{s^2 + \theta^2}. \quad (19) \end{aligned}$$

Subjecting equation (19) for further simplification yields

$$\begin{aligned} & y_i(s) = \frac{P_o}{(d_1 - d_2)} \left(\frac{s}{s^2 + \theta^2} \cdot \frac{1}{s - d_1} \right. \\ & \left. - \frac{s}{s^2 + \theta^2} \cdot \frac{1}{s - d_2} \right) \quad (20a) \end{aligned}$$

where

$$\begin{aligned} & P_o = \left(\frac{j\pi}{L} \right), \quad \theta = \left(\frac{j\pi v t}{L} \right), \\ & d_1 = \frac{-b_2 + \sqrt{b_2^2 - 4b_1 b_3}}{2b_1} \end{aligned}$$

$$d_2 = \frac{-b_2 - \sqrt{b_2^2 - 4b_1b_3}}{2b_1} \quad (20b)$$

To obtain the Laplace inversion of equation (20a), we shall adopt the following representations

$$g(s) = \frac{s}{s^2 + \theta^2}, f_1(s) = \frac{1}{s - d_1} \text{ and} \\ f_2(s) = \frac{1}{s - d_2} \quad (20c)$$

So that the Laplace inversion of each term of the RHS (20a) is the convolution of f_i 's and g defined by as

$$f_s * g = \int_0^t f_i(t-u)g(u)du, \quad i = 1,2 \quad (20d)$$

Thus the Laplace inversion of (20a) is given by

$$y_i(t) = P_k \left[\frac{e^{d_1 t}}{d_1} I_8 - \frac{e^{d_2 t}}{d_2} I_9 \right] \quad (20e)$$

where

$$I_8 = \int_0^t e^{-d_1 u} \cos \theta u du,$$

$$I_9 = \int_0^t e^{-d_2 u} \cos \theta u du \quad \text{and}$$

$$P_k = \frac{P_0}{(d_1 - d_2)} \quad (20f)$$

Further simplification of equation (20e) and substituting into equation (13) which on inversion yields

$$z(x,t) = \sum_{k=1}^n P_k \left[\frac{e^{d_1 t}}{d_1(\theta^2 + d_1^2)} \right. \\ \times (d_1 - \theta e^{d_1 t} \sin \theta t - d_1 e^{d_1 t} \cos \theta t) \\ \left. - \frac{e^{d_2 t}}{d_2(\theta^2 + d_2^2)} (d_2 - \theta e^{d_2 t} \sin \theta t \right. \\ \left. - d_2 e^{d_2 t} \cos \theta t) \right] \sin \frac{i\pi x}{L}. \quad (21)$$

Equation (21) represents the transverse displacement response of the non prismatic Bernoulli beam resting on exponentially decaying

foundation under the action of fast moving distributed forces.

Case II:

Response of non-prismatic beam to harmonic variable magnitude moving distributed loads.

The dynamic behavior of structurally damped non prismatic beam when subjected to harmonic variable magnitude moving load is investigated in this section. Thus, the load $P(x,t)$ is given as

$$P(x,t) = P \sin \Omega t H(x - v_m t) \quad (22)$$

where Ω is the circular frequency of the harmonic force and all parameters are as defined previously. Substituting equation (22) in equation (1), vibration of the beam is then described by the equation

$$-q_x(x,t) + \mu(x)z_{tt}(x,t) + b(x)z_t(x,t) \\ - \mu(x)R_{o,z_{xxx}}(x,t) - Nz_{xx}(x,t) - Gz_{xx}(x,t) \\ + F(x)z(x,t) = PH(x - v_m t) \sin \Omega t. \quad (23)$$

Equation (23) is the governing equation describing the motion of non- prismatic beams subjected to fast moving loads of varying magnitude. Like in the previous section, as closed-form solution to equation (23) is sought.

$$b_1(m,j)\ddot{y}_m(t) + b_2(m,j)\dot{y}_m(t) + b_3(m,j)y_m(t) \\ = P_o \sin \Omega t \cos \theta \quad (24)$$

Equation (24) is analogous to equation (18). Subjecting equation (24) to Laplace transform in conjunction with the boundary conditions stated (5) and using convolution theory yields

$$y_b(x,t) = \sum_{k=1}^{\infty} P_k \left[\frac{e^{d_1 t} \eta_1}{d_1(\eta_1^2 + d_1^2)} \right. \\ \times \left(1 - \frac{d_1}{\eta_1} e^{d_1 t} \sin \eta_1 t - e^{d_1 t} \cos \eta_1 t \right) \\ \left. + \frac{e^{d_2 t} \eta_2}{d_1(\eta_2^2 + d_1^2)} \right]$$

$$\begin{aligned} & \times \left(1 - \frac{d_1}{\eta_2} e^{d_1 t} \sin \eta_2 t - e^{d_1 t} \cos \eta_2 t \right) \\ & - \frac{e^{d_2 t} \eta_1}{d_2 (\eta_1^2 + d_2^2)} \\ & \times \left(1 - \frac{d_2}{\eta_1} e^{d_2 t} \sin \eta_1 t - e^{d_2 t} \cos \eta_1 t \right) \\ & - \frac{e^{d_2 t} \eta_2}{d_2 (\eta_2^2 + d_2^2)} \\ & \times \left(1 - \frac{d_2}{\eta_2} e^{d_2 t} \sin \eta_2 t - e^{d_2 t} \cos \eta_2 t \right) \Big] \sin \frac{m\pi x}{L} \end{aligned} \quad (25a)$$

where

$$\eta_1 = \Omega + \theta \text{ and } \eta_2 = \Omega - \theta \quad (25b)$$

Equation (25a) represents the transverse displacement response of non prismatic beam resting on exponentially decaying foundation under actions of harmonic variable magnitude moving distributed loads.

DISCUSSION ON THE CLOSED FORM SOLUTION

The transverse displacement of an elastic beam may increase without bound. Thus one is interested in resonance conditions. Equation (21) clearly depicts that the beam resting on exponentially decaying foundation traverse by a constant moving load will grow without bound whenever

$$\begin{aligned} d_1 &= d_2, & d_1^2 &= -\theta^2 \\ \text{or } d_2^2 &= -\theta^2 \end{aligned} \quad (26)$$

and the velocity at which this occurs, known as critical velocity is given by the relation

$$c_i = \left(\frac{L}{a_1 k \pi} \right) \sqrt{\frac{1}{2} \left((a_2^2 - 4a_1 a_2)^{1/2} + 2a_1 a_3 - a_2^2 \right)} \quad (27)$$

While equation (25a) shows that the same beam under the action of harmonic variable magnitude

moving loads will experience resonance effects whenever

$$\begin{aligned} d_1 &= d_2, & d_1^2 &= -\eta_1^2 \\ \text{or } d_2^2 &= -\eta_1^2 \end{aligned} \quad (28)$$

and the velocity at which this occurs, known as critical velocity is given by the relation

$$c_m = \sqrt{\left((a_2^2 - 4a_1 a_2)^{1/2} + 2a_1 a_3 - a_2^2 \right) \frac{1}{2} \left(\frac{L}{a_1 k \pi} \right)^2 - \sqrt{\Omega}} \quad (29)$$

Therefore, it is evident from equations (27) and (29) that the critical velocity of non prismatic beam and under the actions of constant magnitude moving load is greater than that of the same beam under the action variable magnitude moving load. Hence, resonance is reached earlier in the latter.

Remarks on Numerical Results

For the foregoing analysis, the non prismatic beam of length is taken to be 12.20m. Furthermore, flexural rigidity EI is $6.068 \times 10^6 \text{m}^3/5^2$, $\gamma = 0.025$ and the moving load is assumed to travel at the constant velocity of 8.123m/s. The transverse deflections of the beam are calculated and plotted against time for various values of foundation constant (moduli) and spatial coordinates x. Values of F_s varied between 0 N/m^3 and 500000 N/m^3

Figure 1: Transverse displacement response of a beam subjected to constant magnitude moving loads for various values of R_s , and for fixed values of $F_s = 4000$ and $N=200000$

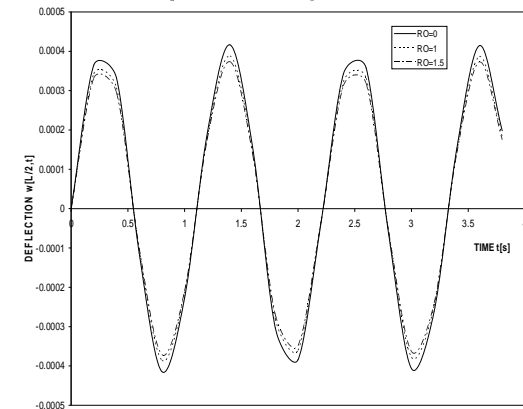


Figure 2: Displacement response of a beam subjected to constant magnitude moving loads for various values of F_s and for fixed values of $R_o = 0.5$ and $N=4000$

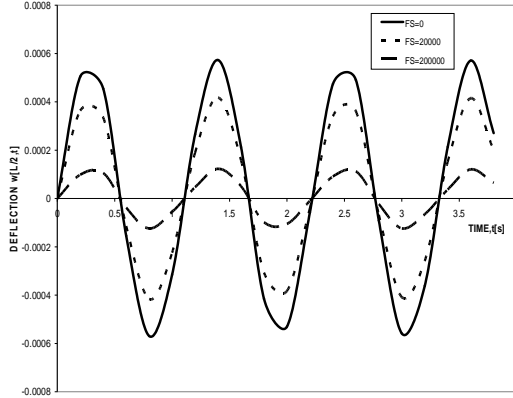


Figure 3: Transverse displacement response of a beam subjected to constant magnitude moving loads for various values of axial force N and for fixed value of $F_s = 4000$ and $R_o = 0.5$

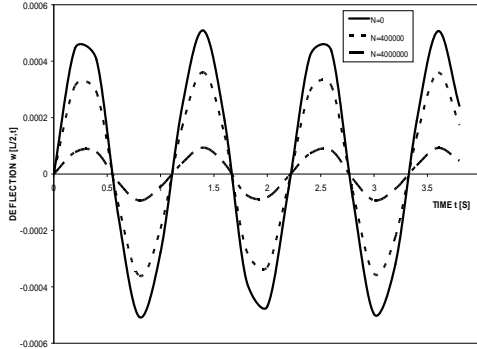


Figure 4: Deflection profile of a beam subjected to Harmonic variable magnitude moving loads for various values of Rotatory inertia R_o

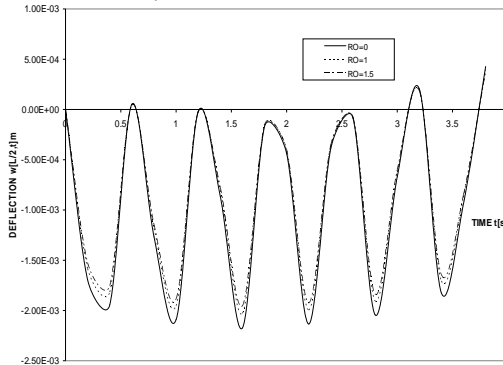


Figure 5: Transverse displacement response of a beam subjected to Harmonic variable magnitude moving loads for various values of F_s and for fixed value of $N=4000$ and $R_o = 0.5$

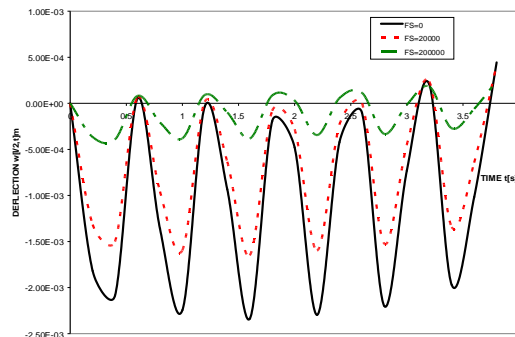


Figure 6: Transverse displacement response of a beam subjected to Harmonic variable magnitude moving loads for various values of N and for fixed value of $F_s = 4000$ and $R_o = 0.5$

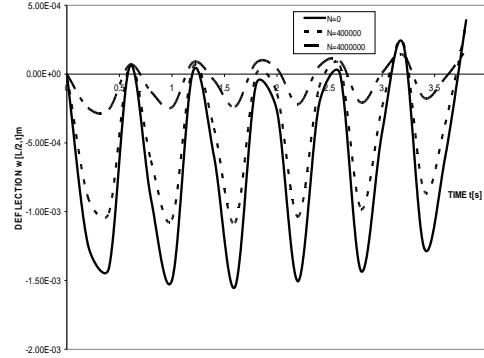
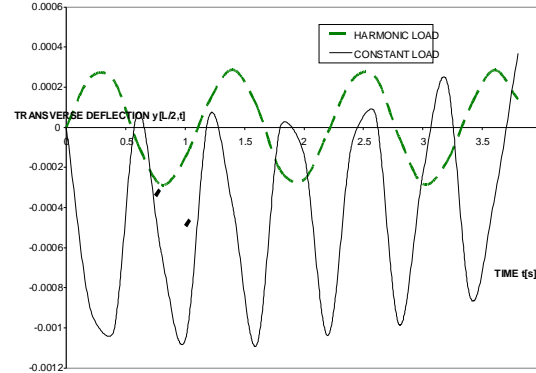


Figure 7: comparison of the traversed displacement of constant and harmonic variable moving loads for fixed values $F_s=4000$, $R_o = 0.5$ and $N=4000$.



In figures 1 and 4, the deflection profile of a simply supported non prismatic Rayleigh beam under the action of constant magnitude and variable harmonic magnitude moving loads forces at constant velocity for various values of R_o and for fixed values of $F_s=4000N/m^3$, and $N=200000N/m^3$ is displayed. The figures show that as R_o increases, the deflection profiles of the non prismatic Rayleigh beam decrease.

Figures 2 and 5 display the deflection profile of the simply supported beam under the action of traveling distributed forces when traveling loads are of constant magnitude and variable harmonic magnitude. For various values of F_s , The figures show that as F_s increases the deflection of the non prismatic beam decreases. Also for various time t , the displacement of the beam for fixed values of R_o and F_s , for various values of N , are shown in figures 3 and 6. It is seen from these figures that as the values of N increases, the transverse displacement of the simply supported beam under the action of both moving constant and harmonic variable moving loads decreases.

Finally, figure 7 depicts the comparison of the traversed displacement of constant and harmonic variable moving loads for fixed values $F_s = 4000$,

$R_0 = 0.5$ and $N=4000$. Clearly, the response amplitude of variable magnitude moving load is higher than that of the constant magnitude moving load.

CONCLUSION

The dynamic behaviour of a beam resting on non-Winkler foundation exponentially decaying foundation when carrying fast traveling distributed loads of constant and varying magnitudes has been investigated. The versatile analytical technique known as Galerkin's method has been employed in conjunction with integral transform method to obtain closed-form solution of this dynamical beam-load problem. Numerical analysis is also carried out and results show the following:

- a. As the value of prestress N increases, the displacement response of a non prismatic Rayleigh beam decreases for fixed values of foundation modulus, F_s and Rotatory inertia R_0 .
- b. For the constant and variable magnitude moving loads problems, the response amplitudes of the beam decrease with an increase in the foundation constant F_s .
- c. As the value of R_0 increases, the displacement responses of both constant and harmonic variable moving loads decrease for fixed values of F_s and N .
- d. The responses amplitude of the harmonic variable load is smaller than that of the constants magnitude load. This shows that resonance is reached earlier in constant magnitude problem.

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