

## Analytical Solution of a Mathematical Model for the Dynamics of Diabetes Mellitus and its Complications incorporating Treatment and Positive Lifestyle as Control

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### Abstract

#### Keywords:

Diabetes mellitus, complications, treatment, positive lifestyle.

*In this paper, a mathematical model for the dynamics of diabetes mellitus and its complications incorporating treatment and positive lifestyle as control is developed. The population under study is divided into healthy, susceptible, diabetic without complication, diabetic with complication, diabetic with complications undergoing treatment. The analytical solution of the model is obtained using homotopy perturbation method (HPM). We carry out numerical simulation of the solution and the graphical results are presented. The result of the study established that the disease can be controlled by effective treatment and positive lifestyle.*

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### INTRODUCTION

Diabetes is a disorder in which the body becomes unable to control the amount of sugar in the blood. The pancreas (beta-cells) is not functioning normally, resulting in a partial or total lack of insulin which is the key to the mechanism converting sugar to energy. Consequently, the system is no longer self-regulated and the level of sugar in the blood goes beyond the limits (WHO and IDF, 2004). Diabetes prevalence is rising dramatically worldwide and is expected to rise from 366 million in 2011 to 552 million by 2030 (Wild et al., 2000). More than 10% of world health care expenditure and about 14% of U.S. healthcare costs are attributable to diabetes (WHO, 2014). Diabetes is sweeping the globe as a silent epidemic largely contributing to the growing burden of non-communicable diseases and mainly encourage by decreasing level of physical activities, poor dieting

and unhealthy lifestyle. It is the sixth leading cause of deaths in the world as reported by WHO. In order to reduce the burden of diabetes and deaths due to complications arising from diabetes, we develop a robust model incorporating treatment and positive lifestyle as measures for controlling the evolution of incidence of diabetes and occurrence of complications of diabetes. Previous research work on diabetes and its complications (IDF, 2014; Adamu et al., 2016; Adewale et al., 2007; Akinsola and Oluyo, 2014; Boutayed et al., 2004; Boutayeb et al., 2006; Padma and Deepa, 2012; Permatasari et al., 2018), were formulated on the belief that the cause of diabetes is genetic disorder. In this work, we consider genetic factors and lifestyle as causes of diabetes and show the effectiveness of the control measures

**MATERIALS AND METHODS**

**The Mathematical Model**

The model equations are formulated using first order ordinary differential equations. Improving on the work of Adamu *et al.*(2016)

and Akinsola *et al.*(2014), we introduced a mathematical model of diabetes and its complications incorporating treatment and positive lifestyle as control

$$\left. \begin{aligned} \frac{dH}{dt} &= \beta\theta - (\mu + \tau)H + \sigma S \\ \frac{dS}{dt} &= \beta(1 - \theta) + \tau H - (\mu + \alpha + \sigma)S \\ \frac{dD}{dt} &= \alpha S - \mu D - \lambda(1 - \phi)D + \omega T \\ \frac{dC}{dt} &= \lambda(1 - \phi)D - (\mu + \nu + \delta + \gamma)C \\ \frac{dT}{dt} &= \gamma C - (\mu + \omega)T \end{aligned} \right\} (2.1)$$

Variables	Description
$H(t)$	healthy class
$S(t)$	susceptible class
$D(t)$	diabetic without complications class
$C(t)$	diabetics with complications class
$T(t)$	diabetics with complications undergoing treatment class

Parameters	Description
$\alpha$	probability rate of incidence of diabetes
$\beta$	birth rate
$\lambda$	rate of a diabetic person developing a complication
$\gamma$	rate at which diabetic with complication are treated
$\mu$	natural mortality deaths
$\nu$	rate at which diabetic patients with complication become severely disabled
$\delta$	mortality rate due to complication
$\theta$	proportion of children born into healthy class
$\sigma$	rate at which susceptible individual become healthy
$\tau$	rate at which healthy individual become susceptible
$\omega$	rate at which diabetic with complication after treatment return to diabetic without complications
$\phi$	measure of effective management of diabetes
t	time as continuous independent variable

**Homotopy Perturbation Method**

In recent years, the application of the Homotopy Perturbation Method (HPM) in non-linear problems has been developed by scientists and engineers, because this method continuously deforms the difficult problem under study into a simple problem which is easier to solve. The homotopy perturbation method was proposed first by He in He, (1999) and was developed and improved by He, (2000a), and He, (2004a). Homotopy, is an important part of differential topology. Actually the homotopy perturbation method is a coupling of the traditional perturbation method and the homotopy method in

topology He, (2000b). The homotopy perturbation method (HPM) provides an approximate analytical solution in a series form. HPM has been widely used by numerous researchers successfully for different physical systems such as, bifurcation, asymptotology, nonlinear wave equations, oscillators with discontinuities by (He, 2004a), reaction-diffusion equation and heat radiation equation by (Ganji and Rajabi, 2006), Approximate Solution of SIR Infectious Disease Model (Abubakar *et al.*, 2013) and solution of mathematical study of diabetes model (Enagi *et al.*, 2017)

**Analytical Solution by the Homotopy Perturbation Method (HPM)**

Equations (2.1) can be written as

$$\left. \begin{aligned} \frac{dH}{dt} + k_1 H - \sigma S - \beta\theta &= 0 \\ \frac{dS}{dt} - \tau H + k_2 S - \beta(1 - \theta) &= 0 \\ \frac{dD}{dt} - \alpha S + k_3 D - \omega T &= 0 \\ \frac{dC}{dt} - \lambda D + \lambda\phi D + k_4 C &= 0 \\ \frac{dT}{dt} - \gamma C + k_5 T &= 0 \end{aligned} \right\} \quad (3.1)$$

where

$$k_1 = \mu + \tau, \quad k_2 = \mu + \alpha + \sigma, \quad k_3 = \mu + \lambda - \lambda\phi, \\ k_4 = \mu + \nu + \delta + \gamma, \quad k_5 = \mu + \omega$$

With the initial conditions  $H(0) = H_0$ ,

$$S(0) = S_0, \quad D(0) = D_0, \quad C(0) = C_0 \quad \text{and} \quad T(0) = T_0$$

Let

$$H(t) = m_0 + p m_1 + p^2 m_2 + \dots \quad (3.2)$$

$$S(t) = n_0 + p n_1 + p^2 n_2 + \dots \quad (3.3)$$

$$D(t) = x_0 + p x_1 + p^2 x_2 + \dots \quad (3.4)$$

$$C(t) = y_0 + p y_1 + p^2 y_2 + \dots \quad (3.5)$$

$$T(t) = z_0 + p z_1 + p^2 z_2 + \dots \quad (3.6)$$

Applying HPM to the first equation of (3.1) gives

$$(1-p) \frac{dH}{dt} + p \left[ \frac{dH}{dt} + k_1 H - \sigma S - \beta\theta \right] = 0 \quad (3.7)$$

Substituting (3.2) to (3.3) into (3.7) and simplifying gives

$$m'_0 + p m'_1 + p^2 m'_2 + \dots \\ + p k_1 (m_0 + p m_1 + p^2 m_2 + \dots) \\ - p \sigma (n_0 + p n_1 + p^2 n_2 + \dots) \\ - p \beta \theta = 0 \quad (3.8)$$

$$\left. \begin{aligned} p^0 : m'_0 &= 0 \\ p^1 : m'_1 + k_1 m_0 - \sigma n_0 - \beta\theta &= 0 \\ p^2 : m'_2 + k_1 m_1 - \sigma n_1 &= 0 \end{aligned} \right\} \quad (3.9)$$

Applying HPM to the second equation of (3.1) gives

$$(1-p) \frac{dS}{dt} + p \left[ \frac{dS}{dt} - \tau H + k_2 S + \beta\theta - \beta \right] = 0 \quad (3.10)$$

Substituting (3.2) to (3.3) into (3.10) and simplify give

$$n'_0 + p n'_1 + p^2 n'_2 + \dots \\ - p \tau (m_0 + p m_1 + p^2 m_2 + \dots) \\ + p k_2 (n_0 + p n_1 + p^2 n_2 + \dots + \beta\theta - \beta) = 0 \quad (3.11)$$

Collecting the coefficients of powers of P in (3.11) gives

$$\left. \begin{aligned} p^0 : n'_0 &= 0 \\ p^1 : n'_1 - \tau m_0 + k_2 n_0 + \beta\theta - \beta &= 0 \\ p^2 : n'_2 - \tau m_1 + k_2 n_1 &= 0 \end{aligned} \right\} \quad (3.12)$$

Applying HPM to the third equation of (3.1) gives

$$(1-p) \frac{dD}{dt} + p \left[ \frac{dD}{dt} - \alpha S + k_3 D - \omega T \right] = 0 \quad (3.13)$$

Substituting (3.3),(3.4) and (3.6) into (3.13) and simplify gives

$$x'_0 + p x'_1 + p^2 x'_2 + \dots \\ - p \alpha (n_0 + p n_1 + p^2 n_2 + \dots) \\ + p k_3 (x_0 + p x_1 + p^2 x_2 + \dots) \\ - p \omega (z_0 + p z_1 + p^2 z_2 + \dots) = 0 \quad (3.14)$$

Collecting the coefficients of the powers of P in (3.14) gives

$$\left. \begin{aligned} p^0 : x'_0 &= 0 \\ p^1 : x'_1 - \alpha n_0 + k_3 x_0 - \omega z_0 &= 0 \\ p^2 : x'_2 - \alpha n_1 + k_3 x_1 - \omega z_1 &= 0 \end{aligned} \right\} \quad (3.15)$$

Applying HPM to the fourth equation of (3.1) gives

$$(1-p)\frac{dC}{dt} + p\left[\frac{dC}{dt} - \lambda D + \lambda\phi D + k_4 C\right] = 0 \quad (3.16)$$

Substituting (3.4) to (3.5) into (3.16) and simplify gives

$$\begin{aligned} & y'_0 + p y'_1 + p^2 y'_2 + \dots \\ & - p\lambda(x_0 + p x_1 + p^2 x_2 + \dots) \\ & + p\lambda\phi(x_0 + p x_1 + p^2 x_2 + \dots) \\ & + p k_4(y_0 + p y_1 + p^2 y_2 + \dots) = 0 \end{aligned} \quad (3.17)$$

Collecting the coefficients of the powers of P

in (3.17) gives

$$\left. \begin{aligned} p^0 : y'_0 &= 0 \\ p^1 : y'_1 - \lambda x_0 + \lambda\phi x_0 + k_4 y_0 &= 0 \\ p^2 : y'_2 - \lambda x_1 + \lambda\phi x_1 + k_4 y_1 &= 0 \end{aligned} \right\} \quad (3.18)$$

Applying HPM to the fifth equation of (3.1) gives

$$(1-p)\frac{dT}{dt} + p\left[\frac{dT}{dt} - \gamma C + k_5 T\right] = 0 \quad (3.19)$$

Substituting (3.5) to (3.6) into (3.19) and simplify gives

$$\begin{aligned} & z'_0 + p z'_1 + p^2 z'_2 + \dots \\ & - p\gamma(y_0 + p y_1 + p^2 y_2 + \dots) \\ & + p k_5(z_0 + p z_1 + p^2 z_2 + \dots) = 0 \end{aligned} \quad (3.20)$$

Collecting the coefficients of powers of P in (3.20) gives

$$\left. \begin{aligned} p^0 : z'_0 &= 0 \\ p^1 : z'_1 - \gamma y_0 + k_5 z_0 &= 0 \\ p^2 : z'_2 - \gamma y_1 + k_5 z_1 &= 0 \end{aligned} \right\} \quad (3.21)$$

Substituting (3.22), (3.24) and (3.25) into (3.3) gives

Solving the first equations of (3.9), (3.12), (3.15), (3.18) and (3.21) gives

$$\left. \begin{aligned} m_0 &= H_0 \\ n_0 &= S_0 \\ x_0 &= D_0 \\ y_0 &= C_0 \\ z_0 &= T_0 \end{aligned} \right\} \quad (3.22)$$

Substituting (3.22) into second equations of (3.9), (3.12), (3.15), (3.18) and (3.21) and solving gives

$$\left. \begin{aligned} m_1 &= (\beta\theta - k_1 H_0 + \sigma S_0) \\ n_1 &= (\tau H_0 - k_2 S_0 - \beta\theta + \beta) \\ x_1 &= (\alpha S_0 - k_3 D_0 + \omega T_0) \\ y_1 &= (\lambda D_0 - \lambda\phi D_0 - k_4 C_0) \\ z_1 &= (\gamma C_0 - k_5 T_0) \end{aligned} \right\} \quad (3.24)$$

Substituting (3.24) into the third equations of (3.9), (3.12), (3.15), (3.18) and (3.21) gives

$$\left. \begin{aligned} m_2 &= \left[ \frac{\sigma(\tau H_0 - k_2 S_0 - \beta\theta + \beta)}{-k_1(\beta\theta - k_1 H_0 + \sigma S_0)} \right] \frac{t^2}{2} + \dots \\ n_2 &= \left[ \frac{\tau(\beta\theta - k_1 H_0 + \sigma S_0)}{-k_2(\tau H_0 - k_2 S_0 - \beta\theta + \beta)} \right] \frac{t^2}{2} + \dots \\ x_2 &= \left[ \frac{\alpha(\tau H_0 - k_2 S_0 - \beta\theta + \beta)}{-k_3(\alpha S_0 - k_3 D_0 + \omega T_0) + \omega(\gamma C_0 - k_5 T_0)} \right] \frac{t^2}{2} + \dots \\ y_2 &= \left[ \frac{\lambda(\alpha S_0 - k_3 D_0 + \omega T_0)}{-\lambda\phi(\alpha S_0 - k_3 D_0 + \omega T_0) - k_4(\lambda D_0 - \lambda\phi D_0 - k_4 C_0)} \right] \frac{t^2}{2} + \dots \\ z_2 &= \left[ \frac{\gamma(\lambda D_0 - \lambda\phi D_0 - k_4 C_0)}{-k_5(\gamma C_0 - k_5 T_0)} \right] \frac{t^2}{2} + \dots \end{aligned} \right\} \quad (3.25)$$

Substituting (3.22), (3.24) and (3.25) into (3.2) gives

$$\begin{aligned} H(t) &= H_0 + p[(\beta\theta - k_1 H_0 + \sigma S_0)] \\ & + p^2 \left[ \frac{\sigma(\tau H_0 - k_2 S_0 - \beta\theta + \beta)}{-k_1(\beta\theta - k_1 H_0 + \sigma S_0)} \right] \frac{t^2}{2} + \dots \end{aligned} \quad (3.26)$$

$$H(t) = \lim_{p \rightarrow 1} m = m_0 + m_1 + m_2 \quad (3.27)$$

$$\begin{aligned} H(t) &= H_0 + (\beta\theta - k_1 H_0 + \sigma S_0) \\ & - \left[ \frac{\sigma(\tau H_0 - k_2 S_0 - \beta\theta + \beta)}{-k_1(\beta\theta - k_1 H_0 + \sigma S_0)} \right] \frac{t^2}{2} + \dots \end{aligned} \quad (3.28)$$

Substituting (3.22), (3.24) and (3.25) into (3.3) gives

$$S(t) = S_0 + p[(\tau H_0 - K_2 S_0 - \beta\theta + \beta)t] + p^2 \left[ \begin{matrix} \tau(\beta\theta - k_1 H_0 + \sigma S_0) \\ -k_2(\tau H_0 - k_2 S_0 - \beta\theta + \beta) \end{matrix} \right] \frac{t^2}{2} + \dots \quad (3.29)$$

$$S(t) = \lim_{p \rightarrow 1} n = n_0 + n_1 + n_2 \quad (3.30)$$

$$S(t) = S_0 + (\tau H_0 - k_2 S_0 - \beta\theta + \beta)t + \left[ \begin{matrix} \tau(\beta\theta - k_1 H_0 + \sigma S_0) \\ -k_2(\tau H_0 - k_2 S_0 - \beta\theta + \beta) \end{matrix} \right] \frac{t^2}{2} + \dots \quad (3.31)$$

Substituting (3.22), (3.24) and (3.25) into (3.4) gives

$$D(t) = D_0 + p[(\alpha S_0 - k_3 D_0 + \omega T_0)t] + p^2 \left[ \begin{matrix} \alpha(\tau H_0 - k_2 S_0 - \beta\theta + \beta) \\ -k_3(\alpha S_0 - k_3 D_0 + \omega T_0) \\ +\omega(\gamma C_0 - k_5 T_0) \end{matrix} \right] \frac{t^2}{2} + \dots \quad (3.32)$$

$$D(t) = \lim_{p \rightarrow 1} x = x_0 + x_1 + x_2 \quad (3.33)$$

$$D(t) = D_0 + (\alpha S_0 - k_3 D_0 + \omega T_0)t + \left[ \begin{matrix} \alpha(\tau H_0 - k_2 S_0 - \beta\theta + \beta) \\ -k_3(\alpha S_0 - k_3 D_0 + \omega T_0) \\ +\omega(\gamma C_0 - k_5 T_0) \end{matrix} \right] \frac{t^2}{2} + \dots \quad (3.34)$$

Substituting (3.22), (3.24) and (3.25) into (3.5) gives

$$C(t) = C_0 + p[(\lambda D_0 - \lambda\phi D_0 - k_4 C_0)t] + p^2 \left[ \begin{matrix} \lambda(\alpha S_0 - k_3 D_0 + \omega T_0) \\ -\lambda\phi(\alpha S_0 - k_3 D_0 + \omega T_0) \\ -k_4(\lambda D_0 - k_4 C_0) \end{matrix} \right] \frac{t^2}{2} + \dots \quad (3.35)$$

$$C(t) = \lim_{p \rightarrow 1} y = y_0 + y_1 + y_2 \quad (3.36)$$

$$C(t) = C_0 + (\lambda D_0 - \lambda\phi D_0 - k_4 C_0)t + \left[ \begin{matrix} \lambda(\alpha S_0 - k_3 D_0 + \omega T_0) \\ -\lambda\phi(\alpha S_0 - k_3 D_0 + \omega T_0) \\ -k_4(\lambda D_0 - \lambda\phi D_0 - k_4 C_0) \end{matrix} \right] \frac{t^2}{2} + \dots \quad (3.37)$$

Substituting (3.22), (3.24) and (3.25) into (3.6) gives

$$T(t) = T_0 + p[(\gamma C_0 - k_5 T_0)t] + p^2 \left[ \begin{matrix} \gamma(\lambda D_0 - \lambda\phi D_0 - k_4 C_0) \\ -k_5(\gamma C_0 - k_5 T_0) \end{matrix} \right] \frac{t^2}{2} + \dots \quad (3.38)$$

$$T(t) = \lim_{p \rightarrow 1} z = z_0 + z_1 + z_2 \quad (3.39)$$

$$T(t) = T_0 + (\gamma C_0 - k_5 T_0)t + \left[ \begin{matrix} \gamma(\lambda D_0 - \lambda\phi D_0 - k_4 C_0) \\ -k_5(\gamma C_0 - k_5 T_0) \end{matrix} \right] \frac{t^2}{2} + \dots \quad (3.40)$$

Equations (3.28), (3.31), (3.34), (3.37) and (3.40) are the solutions of (2.1), which are summarized as follows:

$$\left. \begin{aligned}
 H(t) &= H_0 + (\beta\theta - k_1 H_0 + \sigma S_0)t \\
 &+ \left[ \sigma(\tau H_0 - k_2 S_0 - \beta\theta + \beta) \right] \frac{t^2}{2} + \dots \\
 &+ \left[ -k_1(\beta\theta - k_1 H_0 + \sigma S_0) \right] \frac{t^3}{6} + \dots \\
 S(t) &= S_0 + (\tau H_0 - k_2 S_0 - \beta\theta + \beta)t \\
 &+ \left[ \tau(\beta\theta - k_1 H_0 + \sigma S_0) \right] \frac{t^2}{2} + \dots \\
 &+ \left[ -k_2(\tau H_0 - k_2 S_0 - \beta\theta + \beta) \right] \frac{t^3}{6} + \dots \\
 D(t) &= D_0 + (\alpha S_0 - k_3 D_0 + \omega T_0)t \\
 &+ \left[ \alpha(\tau H_0 - k_2 S_0 - \beta\theta + \beta) \right] \frac{t^2}{2} + \dots \\
 &+ \left[ -k_3(\alpha S_0 - k_3 D_0 + \omega T_0) \right] \frac{t^3}{6} + \dots \\
 &+ \left[ \omega(\gamma C_0 - k_5 T_0) \right] \frac{t^3}{6} + \dots \\
 C(t) &= C_0 + (\lambda D_0 - \lambda\phi D_0 - k_4 C_0)t \\
 &+ \left[ \lambda(\alpha S_0 - k_3 D_0 + \omega T_0) \right] \frac{t^2}{2} + \dots \\
 &+ \left[ -\lambda\phi(\alpha S_0 - k_3 D_0 + \omega T_0) \right] \frac{t^2}{2} + \dots \\
 &+ \left[ -k_4(\lambda D_0 - \lambda\phi D_0 - k_4 C_0) \right] \frac{t^3}{6} + \dots \\
 T(t) &= T_0 + (\gamma C_0 - k_5 T_0)t \\
 &+ \left[ \gamma(\lambda D_0 - \lambda\phi D_0 - k_4 C_0) \right] \frac{t^2}{2} + \dots \\
 &+ \left[ -k_5(\gamma C_0 - k_5 T_0) \right] \frac{t^3}{6} + \dots
 \end{aligned} \right\} (3.41)$$

The following initial values of variables are used for the numerical  $H_0 = 177300000$ ,  $S_0 = 90830790$ ,

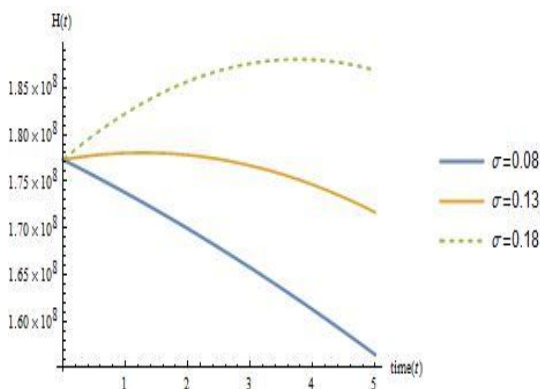
$$D_0 = 940000, C_0 = 3760000, T_0 = 564000$$

The following parameters values are used for the numerical simulation:

$$\begin{aligned}
 \alpha &= 0.05, \beta = 0.01623, \gamma = 0.08, \lambda = 0.05, \\
 \mu &= 0.02, \\
 \nu &= 0.05, \sigma = 0.08, \theta = 0.923, \phi = 0.5, \omega = 0.08, \\
 \delta &= 0.05, \tau = 0.04
 \end{aligned}$$

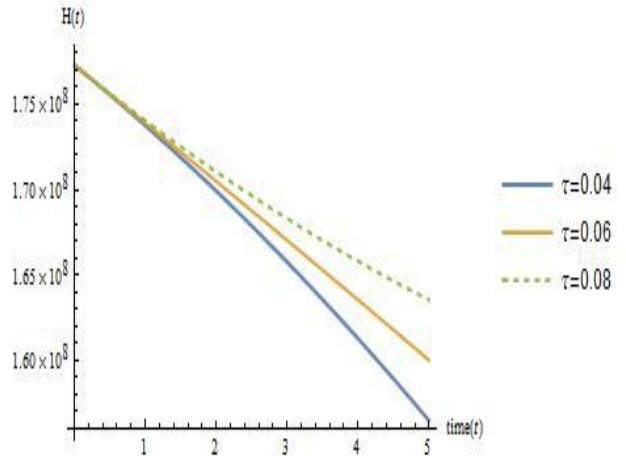
**Numerical Simulation and Result**

We use mathematical software (MATHEMATICA) to carry out the numerical simulations of  $H(t)$ ,  $S(t)$ ,  $D(t)$ ,  $C(t)$  and  $T(t)$  and the results are presented in graphical form.



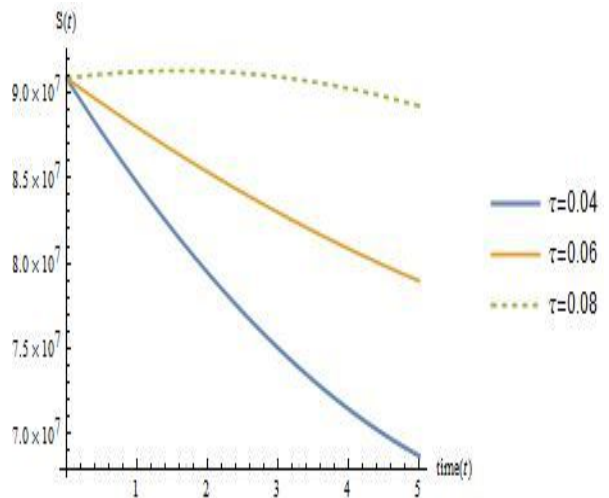
**Figure 4.1: Effect of rate at which Susceptible Individual become Healthy on Healthy Class**

Figure 4.1 shows that, as the rate at which susceptible individual become healthy increases, the healthy class increases faster. This may be attributable to high positive lifestyle such as good dieting and physical exercises. This shows that the more people adopt positive lifestyle, the less they are susceptible to diabetes



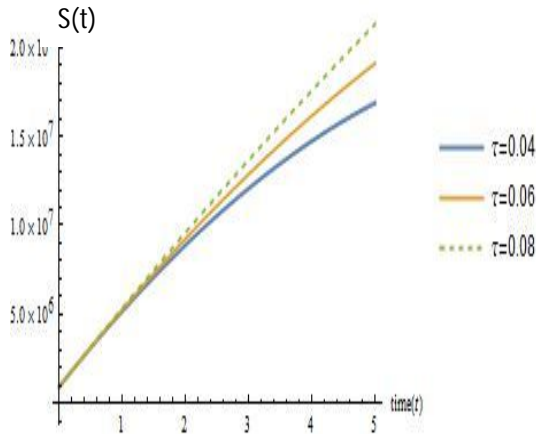
**Figure 4.2: Effect of Susceptibility rate on Healthy Class**

Figure 4.2 shows that, as rate of susceptibility increases, the healthy class increases. This shows that no matter the rate of susceptibility, the healthy class will continue to increase. This may be attributable to high population growth.



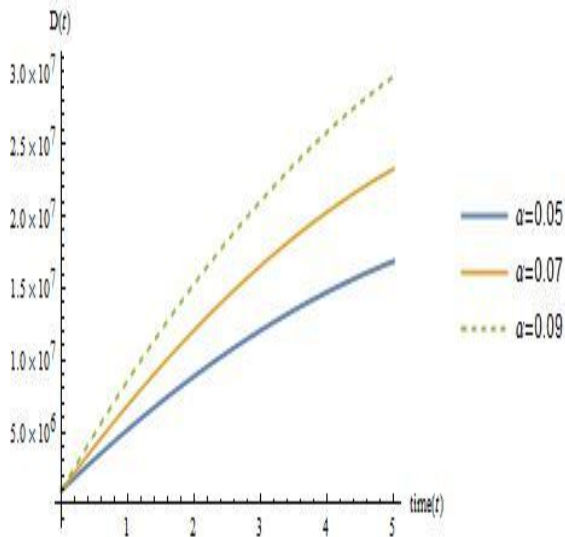
**Figure 4.3: Effect of Susceptibility rate on Susceptible Class**

Figure 4.3 shows that, as susceptibility rate increases, the susceptible class increases faster. This shows that the more people are living unhealthy lifestyle such as smoking, alcoholism, glutoning and lack of physical exercises, the more people are susceptible to diabetes.



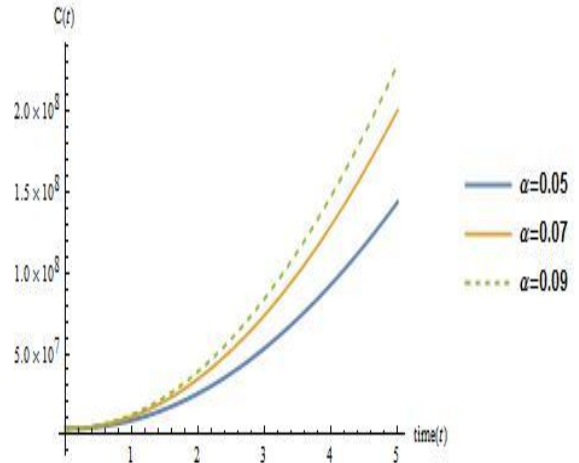
**Figure 4.4: Effect of different Susceptibility rate on Diabetics without Complications**

Figure 4.4 shows that, as the rate of susceptibility increases, diabetics without complications class increases slightly faster. This shows that the more people are susceptible to diabetes, the more incidence of diabetes occur.



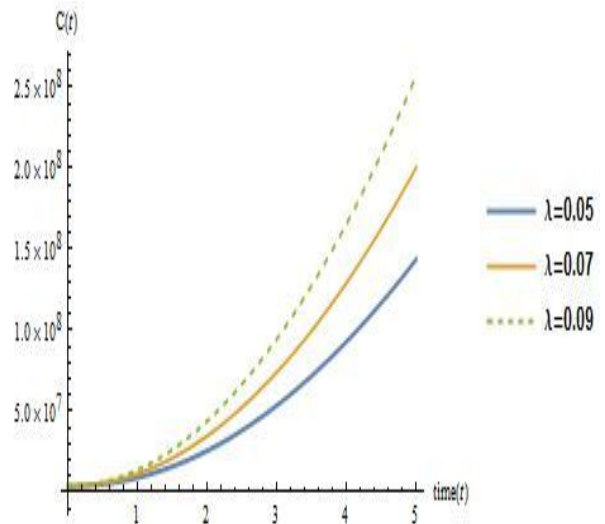
**Figure 4.5: Effect of Probability rate of Incidence of Diabetes on Diabetes without Complications.**

Figure 4.5 shows that, as probability rate of incidence of diabetes increases, diabetics without complications class increases faster. It shows that high rate of incidence of diabetes increases cases of diabetes mellitus.



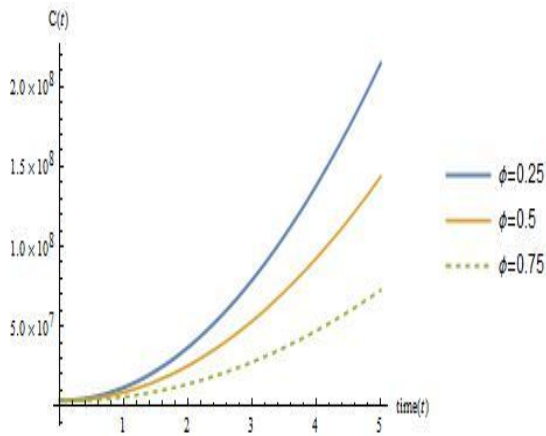
**Figure 4.6: Effect of Probability rate of Incidence of Diabetes on Diabetics with Complications Class.**

Figure 4.6 shows that, as the probability rate of incidence of diabetes increases, diabetics with complications increases faster. This shows that there is link between incidence of diabetes and complications of diabetes.



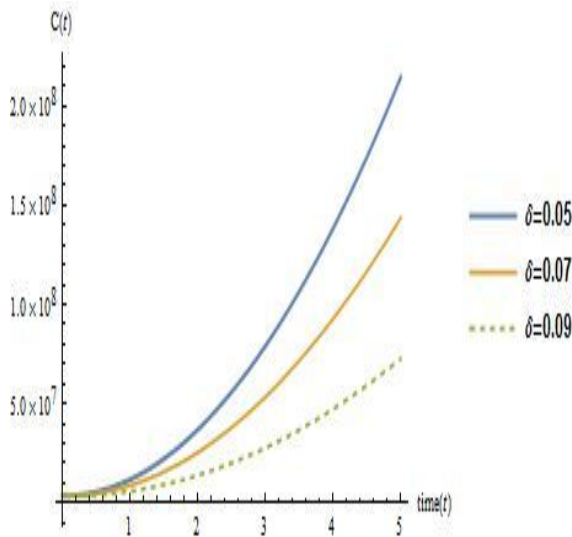
**Figure 4.7: Effect of rate of Diabetic person developing Complications on Diabetics with Complications Class.**

Figure 4.7 shows that, as the rate of diabetic person developing complications increases, diabetics with complications increases faster. Effort can be made to reduce the number of death due to complications of diabetes by controlling the transition from diabetics without complications stage to diabetics with complications stage.



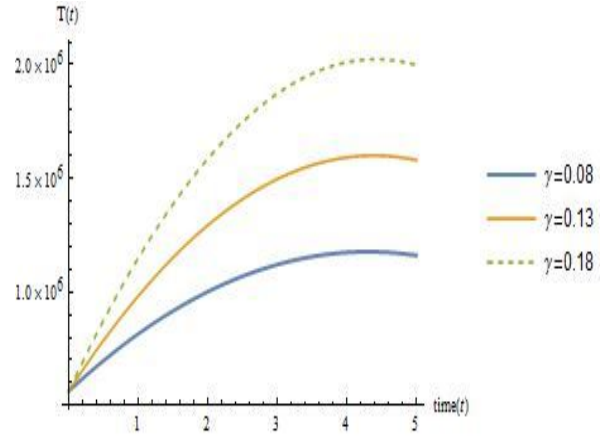
**Figure 4.8: Effect of Control Measures on Diabetics with Complications Class**

Figure 4.8 shows that, as control rate increases, diabetics with complications decreases faster. This shows that the more people adopt positive lifestyle such as good dieting and regular physical exercises, the less the diabetics with complications. It is observed that as the control rate increases, the diabetics with complications are almost zero. However, effort to increase control measure will go a long way to reduce complications arising from diabetes.



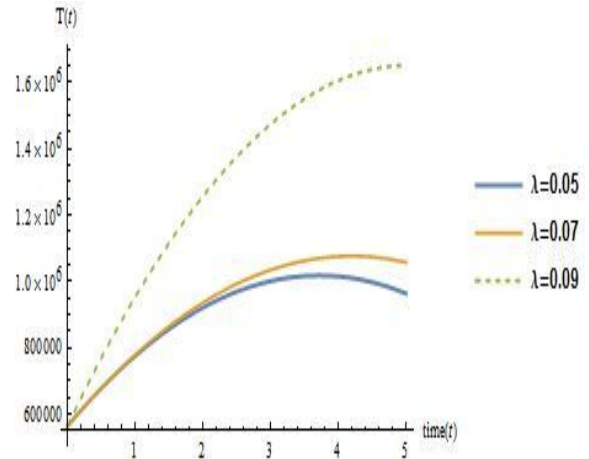
**Figure 4.9: Effect of rate of Death due to Complications on Diabetics with Complications Class.**

Figure 4.9 is the effect of rate of death due to complications. It is shown that high rate of death due to complications of diabetes decreased the diabetics with complications class. This shows that more people die as a result of complications of diabetes when there is no treatment.



**Figure 4.10: Effect of Treatment rate on Diabetics with Complications undergoing Treatment.**

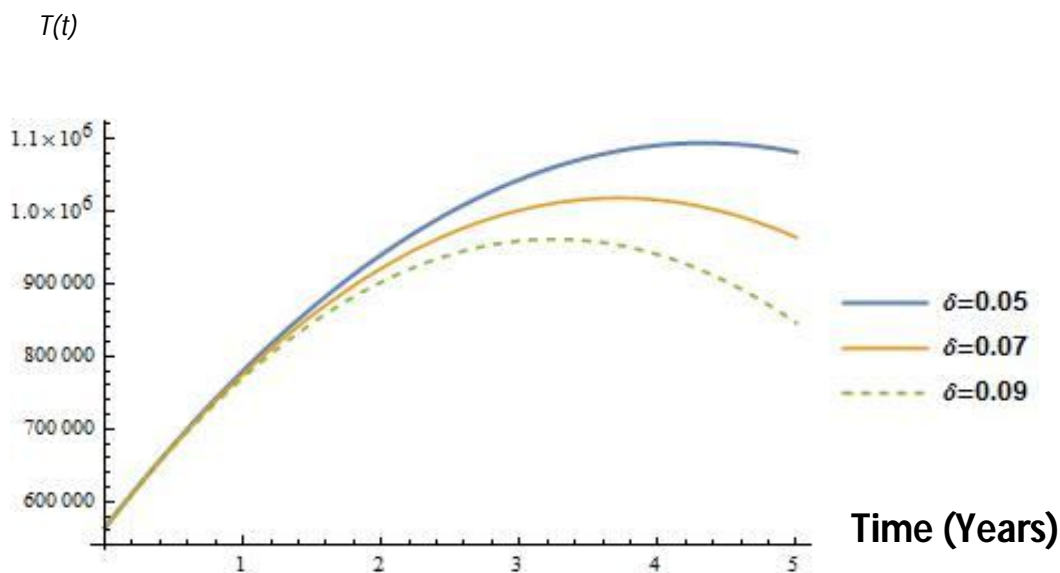
Figure 4.10 is the effect of treatment rate on diabetics with complications undergoing treatment. This shows that high treatment rate increased diabetics with complications undergoing treatment. It is shown that effective treatment helps to controlled the menace of diabetes thereby reducing the number of death due to diabetes.



**Figure 4.11: Effect of rate of Diabetic person Developing Complications on Diabetics with Complications undergoing Treatment.**

Figure 4.11 shows that, as the probability of rate of developing complications increases, diabetics with complications undergoing treatment increases faster. This shows that as more diabetics without complications develop complications, the more the diabetics with complications undergoing treatment. However, it is very important to introduce control measure to reduce the high rate of developing complications.





**Figure 4.12: Effect of rate of Death due to Complications on Diabetics with Complications undergoing Treatment Class.**

Figure 4.12 shows that, as the rate of death due to complications increases, diabetics with complications undergoing treatment decreases faster. This shows that with high rate of death due to complications, diabetics with complications undergoing treatment will reduce to zero.

## 5. Conclusion

We studied mathematical model of diabetes mellitus and its complications incorporating treatment and positive lifestyle as control using Homotopy Perturbation Method and the result obtained shows that parameters involved played a crucial role in the size of healthy class, susceptible class, diabetics without complications class, diabetics with complications class and diabetics with complications undergoing treatment class at time  $t$ .

This research work have established that with adequate control measures such as regular physical activities, good dieting, treatment and reduction in unhealthy lifestyle (smoking, alcoholism and glutoning), prevalence and incidence of diabetes mellitus can be controlled thereby reducing the menace of

diabetes and preventing death due to complications of diabetes.

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