



## Analysis of Mathematical Model for the Dynamics of Lassa Fever

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### Abstract

#### Keywords:

Lassa fever, Differential equation, Homotopy perturbation method, Numerical simulation, Mathematical model.

*In this paper, a mathematical model was proposed to determine the impact of parameters on the population dynamics of Lassa fever infection. The population under study is divided into susceptible, latent, infected, isolated and recovered class. The system of ordinary differential equations describing the dynamics of the disease are derived and analytical solution of the system of equations is obtained using Homotopy Perturbation Method. The numerical simulation of the model was carried out using Maple 18 mathematical software. The parameters are varied and their effects on the disease dynamics are presented graphically. The results showed that the model parameters determine the population dynamics of the disease.*

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### INTRODUCTION

Lassa fever is a viral haemorrhagic fever transmitted by rats. It has been known since the 1950s, but the virus was not identified until 1969, when two missionary nurses died from it in the town of Lassa in the present Borno State of Nigeria. Found predominantly in West Africa (WHO, 2000), it has the potential to cause tens of thousands of deaths. Lassa fever is an animal-borne, or zoonotic, acute viral illness spread by the common African rat. It is endemic in parts of West Africa including Sierra Leone, Liberia, Guinea and Nigeria. Neighboring countries are also at risk because the animal vector lives throughout the region. Lassa fever is caused by a single stranded RNA virus and is a disseminated systemic primary viral infection (Heading and Gopal, 2001; Johnson *et al.*, 1987). The main feature of fatal illness is impaired or delayed cellular immunity leading to fulminant viraemia (Chen and Cosgriff, 2000). The prevalence of antibodies to the virus in the population is 8-52% in Sierra Leone (McCormick *et al.*, 1987), 4-55% in Guinea (Lukashevich *et al.*, 1993), and 21% in Nigeria (McCormick *et al.*,

1987). The natural hosts for the virus are multimammate rats (*Mastomys natalensis*), which breed frequently and are distributed widely throughout west, central, and east Africa (Johnson *et al.*, 1987). They are probably the most common rodent in tropical Africa and are found predominantly in rural areas, and in dwellings more often than in surrounding countryside (McCormick *et al.*, 1987). Members of the genus are infected persistently and shed the virus in their excreta. Humans are infected by contact with the rats or by eating them (they are considered a delicacy and are eaten by up to 90% of people in some areas). Rats found in houses of infected people are seropositive for the virus 10 times more often than those in control houses (Keenlyside *et al.*, 1983). Virus antibodies occur after a febrile illness in twice as many people who eat rats as in those who do not, and deafness (an effect of Lassa fever) occurs four times more frequently (Ter Muelen *et al.*, 1996). Lassa fever presents with symptoms and signs indistinguishable from those of febrile illnesses such as malaria and other viral haemorrhagic fevers such as Ebola. It is difficult to diagnose clinically but should be suspected in patients with fever

( $\geq 38^\circ\text{C}$ ) not responding adequately to antimalarial and antibiotic drugs. The most useful clinical predictors of Lassa fever are fever, pharyngitis, retrosternal pain, and proteinuria for diagnosis; and fever, sore throat, and vomiting for outcome (McCormick *et al.*, 1987). Ribavirin and general support are needed. Ribavirin is almost twice as effective when given intravenously as when taken orally, and if given within six days of the start of illness it may reduce deaths by 90%. Dehydration, oedema, hypotension, and poor renal function are common; fluid replacement or the use of blood transfusion requires careful monitoring (WHO, 2014).

Mathematical modelling has been used effectively to track the transmission dynamics of Lassa fever in human population. Several researchers have employed the knowledge of mathematical modelling to study the transmission dynamics of lassa fever. The work of (Ojo *et al.*, 2021; Akinpelu and Akinwande, 2018; Ojo and Goufo, 2022; Usman and Adamu, 2018) have discussed the dynamics of Lassa fever. Other researchers such as (Ibrahim and Denes, 2021; Faniran, 2017) have also worked on the transmission dynamics of Lassa fever.

In this study, we improved on the work of (Akinpelu and Akinwande, 2018) and incorporate death due to the disease parameter into the isolated class of the proposed model of Lassa fever.

The paper is organized as follows: In Section 2, the compartmental model formulation for the dynamics of Lassa fever is obtained, Section 3 deals with analytical solution of the model equations. Numerical simulation and discussion of results was done in Section 4 while in Section 5, conclusion was made.

## MATERIALS AND METHODS

### Model Formulation

This paper improved on the existing mathematical models of dynamics of Lassa fever infection by incorporating death due to the disease parameter in the isolated class. The human population under study is compartmentalized into susceptible, latent, infected, isolated and recovered class. The model equations are formulated using the first order ordinary differential equations.

$$\left. \begin{aligned} \frac{dS(t)}{dt} &= -\beta S(t) - \mu S(t) \\ \frac{dL(t)}{dt} &= \beta - \gamma L(t) - \mu L(t) - \theta_1 L(t) \\ \frac{dI(t)}{dt} &= \gamma L(t) - \mu I(t) - \delta I(t) - \theta_2 I(t) \\ \frac{dI_s(t)}{dt} &= \theta_1 L + \theta_2 I - \alpha I_s(t) - (\mu + \delta) I_s(t) \\ \frac{dR(t)}{dt} &= \alpha I_s - \mu R \end{aligned} \right\} \quad (1)$$

Parameters	Description
$\beta$	Contact rate
$\alpha$	Recovery rate of the isolated individuals
$\mu$	Natural death rate
$\gamma$	Progression rate of individuals to infection class
$\theta_1$	Rate at which latently infected are isolated due to tracing
$\theta_2$	Rate at which infected individuals are isolated
$\delta$	Death rate due to the disease
$t$	Time as continuous independent variable

Variables	Description
$S(t)$	Susceptible class
$L(t)$	Latent class
$I(t)$	Infected class
$I_s(t)$	Isolated class
$R(t)$	Recovered class

### Homotopy Perturbation Method

The method of homotopy perturbation for solving problems of nonlinear equation was first introduced by Ji Huan (He, 1999) and subsequently, it was improved by (He, 2000a; He, 2000b) and (He, 2006). The method was successfully applied to solve problems made up of nonlinear and linear functional equations. In recent years, scientists, technologists and engineers have applied homotopy perturbation method to reduce problem of linear and non-linear differential equations that seem to be difficult into simpler one that are easier to solve. The method is a combination of homotopy method in topology and traditional perturbation method ((He, 2006)). The method of homotopy perturbation has provided a very convenient approach for obtaining analytical or approximate solution of series of problems in various fields.

Many researchers have successfully applied the method to physical problems such as non-linear wave equations, bifurcation, asymptotology, SIR infection disease model (Abubakar *et al.*, 2013), equation of Schrodinger (Biazar and Ghazvini, 2007), diabetes model (Enagi *et al.*, 2017), heat radiation equation and reaction- diffusion equation (Ganji and Rajabi, 2006; Ganji and Sadighi, 2006), oscillators with discontinuities (He, 2004a; He, 2004b; He, 2005a; He, 2005b), Sine-Gordon and Klein-Gordon equations (Odibat and Momani, 2007; Chowdhury and Hashim, 2007) and equation of diabetes and its complications model (Aye *et al.*, 2021).

### Fundamental Of Homotopy Perturbation Method

The basic illustration of the method according to (He, 2000a), using non-linear differential equation is stated as follows:

$$D(u) - k(x) = 0, \quad x \in \Lambda \quad (2)$$

with boundary condition of:

$$E\left(u, \frac{\partial u}{\partial n}\right) = 0, \quad x \in \Pi \quad (3)$$

$D$  denotes general differential operator, the boundary operator is  $E$ , the known analytical function is  $k(x)$  and  $\Pi$  denotes domain boundary of  $\Lambda$ . The operator  $D$  can be divided into two parts of  $G$  and  $H$ , the linear part is  $G$ , while  $H$  is a non-linear one. Equation (2) can be rewritten as follows:

$$G(u) + H(u) - k(x) = 0, \quad x \in \Lambda$$

The stated HPM equation is as follows:

$$Q(w, r) = (1-r)[G(w) - G(u_0)] + r[G(w) + H(w) - k(x)] = 0 \quad (4)$$

The embedding parameter is  $r \in [0, 1]$  in equation (4) and the first assumption that meets the boundary condition is  $u_0$ . The assumed solution of equation (4) can be expressed in power series as follows:

$$w = w_0 + r w_1 + r^2 w_2 + \dots \quad (5)$$

The correct solution of (2) is obtained by setting  $r = 1$

$$u = \lim_{r \rightarrow 1} w = w_0 + w_1 + w_2 + \dots \quad (6)$$

In most cases, the series (5) converges. The convergent rate depends on the nonlinear operator  $D(w)$  (He, 2000a).

### Analytical solution by the Homotopy Perturbation Method (HPM)

Equations (1) can be written as

$$\left. \begin{aligned} \frac{dS(t)}{dt} + \beta S(t) + \mu S(t) &= 0 \\ \frac{dL(t)}{dt} - \beta + \gamma L(t) + \mu L(t) + \theta_1 L(t) &= 0 \\ \frac{dI(t)}{dt} - \gamma L(t) + \mu I(t) + \delta I(t) + \theta_2 I(t) &= 0 \\ \frac{dI_s(t)}{dt} - \theta_1 L - \theta_2 I + \alpha I_s(t) + (\mu + \delta) I_s(t) &= 0 \\ \frac{dR}{dt} - \alpha I_s + \mu R &= 0 \end{aligned} \right\}$$

$$\left. \begin{aligned} S' + qS &= 0 \\ L' - \beta S + mL &= 0 \\ I' - \gamma L + nI &= 0 \\ I_s' - \theta_1 L - \theta_2 I + rI_s &= 0 \\ R' - \alpha I_s + \mu R &= 0 \end{aligned} \right\} \quad (8)$$

Where

$$q = \beta + \mu, \quad m = (\gamma + \mu + \theta_1), \quad n = \mu + \delta + \theta_2 \quad \text{and} \quad r = \alpha + \mu + \delta$$

With the initial conditions  $S(0) = S_0, L(0) = L_0, I(0) = I_0, I_s(0) = I_{s0}$  and  $R(0) = R_0$

Let

$$S = S_0 + P^1 S_1 + P^2 S_2 + \dots \quad (9)$$

$$L = L_0 + P^1 L_1 + P^2 L_2 + \dots \quad (10)$$

$$I = I_0 + P^1 I_1 + P^2 I_2 + \dots \quad (11)$$

$$I_s = I_{s0} + P^1 I_{s1} + P^2 I_{s2} + \dots \quad (12)$$

$$R = R_0 + P^1 R_1 + P^2 R_2 + \dots \quad (13)$$

Applying HPM to the first equation of (8) gives (14)

$$(1-p)S' + p(S' + qS) = 0 \quad (14)$$

Substituting (9) to (13) into (14) and simplifying gives (15)

$$S_0' + P^1 S_1' + P^2 S_2' + p(q(S_0 + P^1 S_1 + P^2 S_2)) = 0 \quad (15)$$

Collecting the coefficients of power of  $p^{i_s}$  in (15),

we have (16)

$$\left. \begin{aligned} P^0: S_0' &= 0 \\ P^1: S_1' + qS_0 &= 0 \\ P^2: S_2' + qS_1 &= 0 \end{aligned} \right\} \quad (16)$$

Applying HPM to the second equation of (8) gives (17)

$$(1-p)L' + p(L' - \beta S + mL) = 0 \quad (17)$$

Substituting (9) to (13) into (17) and simplifying gives (18)

$$\left. \begin{aligned} (7L_0' - \beta S_0 - \mu L_0) + P^1(L_1' - \beta S_1 + mL_1) + P^2(L_2' - \beta S_2 + mL_2) - \mu(L_0 + P^1 L_1 + P^2 L_2) &= 0 \end{aligned} \right\} \quad (18)$$

Collecting the coefficients of powers of  $p^{i_s}$  in (18), we have (19)

$$\left. \begin{aligned} P^0: L_0' &= 0 \\ P^1: L_1' - \beta S_0 + mL_0 &= 0 \\ P^2: L_2' - \beta S_1 + mL_1 &= 0 \end{aligned} \right\} \quad (19)$$

Applying HPM to the third equation of (8) gives (20)

$$(1-p)I' + p(I' - \gamma L + nI) = 0 \quad (20)$$

Substituting (9) to (13) into (20) and simplifying gives (21)

$$I'_0 + P^1 I'_1 + P^2 I'_2 + p(-\gamma(L_0 + P^1 L_1 + P^2 L_2) + n(I_0 + P^1 I_1 + P^2 I_2)) \quad (21)$$

Collecting the coefficients of powers of  $p^{i\pm}$  in (21), we have (22)

$$\left. \begin{aligned} P^0: I'_0 &= 0 \\ P^1: I'_1 - \gamma L_0 + nI_0 &= 0 \\ P^2: I'_2 - \gamma L_1 + nI_1 &= 0 \end{aligned} \right\} \quad (22)$$

Applying HPM to the fourth equation of (8) gives (23)

$$(1-p)I'_s + p(I'_s - \theta_1 L - \theta_2 I + rI_s) = 0 \quad (23)$$

Substituting (9) to (13) into (23) and simplifying gives (24)

$$I'_{s_0} + P^1 I'_{s_1} + P^2 I'_{s_2} + p(-\theta_1(L_0 + P^1 L_1 + P^2 L_2) - \theta_2(I_0 + P^1 I_1 + P^2 I_2) + r(I_{s_0} + P^1 I_{s_1} + P^2 I_{s_2})) \quad (24)$$

Collecting the coefficients of powers of  $p^{i\pm}$  in (24), we have (25)

$$\left. \begin{aligned} P^0: I'_{s_0} &= 0 \\ P^1: I'_{s_1} - \theta_1 L_0 - \theta_2 I_0 + rI_{s_0} &= 0 \\ P^2: I'_{s_2} - \theta_1 L_1 - \theta_2 I_1 + rI_{s_1} &= 0 \end{aligned} \right\} \quad (25)$$

Applying HPM to the fifth equation of (8) gives (26)

$$(1-p)R' + p(R' - \alpha I_s + \mu R) = 0 \quad (26)$$

Substituting (9) to (13) into (26) and simplifying gives (27)

$$R'_0 + P^1 R'_1 + P^2 R'_2 + p(-\alpha(I_{s_0} + P^1 I_{s_1} + P^2 I_{s_2}) + \mu(R_0 + P^1 R_1 + P^2 R_2)) = 0 \quad (27)$$

Substituting (9) to (13) into (26) and simplifying gives (27)

$$R'_0 + P^1 R'_1 + P^2 R'_2 + p(-\alpha(I_{s_0} + P^1 I_{s_1} + P^2 I_{s_2}) + \mu(R_0 + P^1 R_1 + P^2 R_2)) = 0 \quad (20)$$

(27)

Collecting the coefficients of powers of  $p^{i\pm}$  in (27), we have (28)

$$\left. \begin{aligned} P^0: R'_0 &= 0 \\ P^1: R'_1 - \alpha I_{s_0} + \mu R_0 &= 0 \\ P^2: R'_2 - \alpha I_{s_1} + \mu R_1 &= 0 \end{aligned} \right\} \quad (28)$$

Solving the first equations of (16), (19), (22), (25) and (28) gives

$$\left. \begin{aligned} S_0 &= S_0 \\ L_0 &= L_0 \\ I_0 &= I_0 \\ I_{s_0} &= I_{s_0} \\ R_0 &= R_0 \end{aligned} \right\} \quad (29)$$

Substituting (29) into the second equation of (16), (19), (22), (25) and (28) and

solving gives (30)

$$\left. \begin{aligned} S_1 &= (-qS_0)t \\ L_1 &= (\beta S_0 - mL_0)t \\ I_1 &= (\gamma L_0 - nI_0)t \\ I_{s_1} &= (\theta_1 L_0 + \theta_2 I_0 - rI_{s_0})t \\ R_1 &= (\alpha I_{s_0} - \mu R_0)t \end{aligned} \right\} \quad (30)$$

Substituting (30) into the second equation of (16), (19), (22), (25) and (28) and solving gives (31)

$$\left. \begin{aligned} S_2 &= \frac{-q}{2}(-qS_0)t^2 \\ L_2 &= (\beta(-qS_0) - m(\beta S_0 - mL_0))\frac{t^2}{2} \\ I_2 &= (\gamma(\beta S_0 - mL_0) - n(\gamma L_0 - nI_0))\frac{t^2}{2} \\ I_{s_2} &= (\theta_1(\beta S_0 - mL_0) + \theta_2(\gamma L_0 - nI_0) + r(\theta_1 L_0 + \theta_2 I_0 - rI_{s_0}))\frac{t^2}{2} \\ R_2 &= (\alpha(\theta_1 L_0 + \theta_2 I_0 - rI_{s_0}) - \mu(\alpha I_{s_0} - \mu R_0))\frac{t^2}{2} \end{aligned} \right\} \quad (31)$$

Substituting (29), (30) and (31) into (9), we have (32)

$$S(t) = S_0 + p^1(-qS_0)t + p^2\left(\frac{-q}{2}(-qS_0)t^2\right) \dots$$

Setting  $p = 1$  of (32) becomes

$$S(t) = \lim_{p \rightarrow 1} S = S_0 + S_1 + S_2 \quad (33)$$

Hence,

$$S(t) = S_0 + (-qS_0)t + \left(\frac{-q}{2}(-qS_0)t^2\right) + \dots \quad (34)$$

Substituting (29), (30) and (31) into (10), we have (35)

$$L(t) = L_0 + p^1(\beta S_0 - mL_0)t + p^2\left(\frac{t^2}{2}(\beta(-qS_0) - m(\beta S_0 - mL_0))\right) + \dots \quad (35)$$

Setting  $p = 1$  of (35) becomes

$$L(t) = \lim_{p \rightarrow 1} L = L_0 + L_1 + L_2 \quad (36)$$

Hence,

$$L(t) = L_0 + (\beta S_0 - mL_0)t + \frac{t^2}{2}(\beta(-qS_0) - m(\beta S_0 - mL_0)) + \dots \quad (37)$$

Substituting (29), (30) and (31) into (11), we have (38)

$$I(t) = I_0 + p^1(\gamma L_0 - nI_0)t + p^2\left(\frac{t^2}{2}(\gamma(\beta S_0 - mL_0) - n(\gamma L_0 - nI_0))\right) + \dots \quad (38)$$

Setting  $p = 1$  of (38) becomes

$$I(t) = \lim_{p \rightarrow 1} I = I_0 + I_1 + I_2 \quad (39)$$

Hence,

$$I(t) = I_0 + (\gamma L_0 - nI_0)t + \left(\frac{t^2}{2}(\gamma(\beta S_0 - mL_0) - n(\gamma L_0 - nI_0))\right) + \dots \quad (40)$$

Substituting (29), (30) and (31) into (12), we have (41)

$$I_s(t) = I_{s_0} + p^1(\theta_1 L_0 + \theta_2 I_0 - rI_{s_0})t + p^2\left(\frac{t^2}{2}(\theta_1(\beta S_0 - mL_0) + \theta_2(\gamma L_0 - nI_0) + r(\theta_1 L_0 + \theta_2 I_0 - rI_{s_0}))\right) + \dots \quad (41)$$

Setting  $p = 1$  of (41) becomes

$$I_s(t) = \lim_{p \rightarrow 1} I_s = I_{s_0} + I_{s_1} + I_{s_2} \quad (42)$$

Hence,

$$I_s(t) = I_{s_0} + (\theta_1 L_0 + \theta_2 I_0 - rI_{s_0})t + \left(\frac{t^2}{2}(\theta_1(\beta S_0 - mL_0) + \theta_2(\gamma L_0 - nI_0) + r(\theta_1 L_0 + \theta_2 I_0 - rI_{s_0}))\right) + \dots \quad (43)$$

Substituting (29), (30) and (31) into (13), we have (44)

$$R(t) = R_0 + p^1(\alpha I_{s_0} - \mu R_0)t + p^2\left(\frac{t^2}{2}(\alpha(\theta_1 L_0 + \theta_2 I_0 - rI_{s_0}) - \mu(\alpha I_{s_0} - \mu R_0))\right) + \dots \quad (44)$$

Setting  $p = 1$  of (44) becomes

$$R(t) = \lim_{p \rightarrow 1} R = R_0 + R_1 + R_2 \quad (45)$$

Hence,

$$R(t) = R_0 + (\alpha I_{s_0} - \mu R_0)t + \left(\frac{t^2}{2}(\alpha(\theta_1 L_0 + \theta_2 I_0 - rI_{s_0}) - \mu(\alpha I_{s_0} - \mu R_0))\right) + \dots \quad (46)$$

Equations (34), (37), (40), (41) and (43) are the solutions of (1), which are summarised as follows

$$\left. \begin{aligned} S(t) &= S_0 + (-qS_0)t + \left(\frac{-q}{2}(-qS_0)t^2\right) + \dots \\ L(t) &= L_0 + (\beta S_0 - mL_0)t + \frac{t^2}{2}(\beta(-qS_0) - m(\beta S_0 - mL_0)) + \dots \\ I(t) &= I_0 + (\gamma L_0 - nI_0)t + \left(\frac{t^2}{2}(\gamma(\beta S_0 - mL_0) - n(\gamma L_0 - nI_0))\right) + \dots \\ I_s(t) &= I_{s_0} + (\theta_1 L_0 + \theta_2 I_0 - rI_{s_0})t + \left(\frac{t^2}{2}(\theta_1(\beta S_0 - mL_0) + \theta_2(\gamma L_0 - nI_0) + r(\theta_1 L_0 + \theta_2 I_0 - rI_{s_0}))\right) + \dots \\ R(t) &= R_0 + (\alpha I_{s_0} - \mu R_0)t + \left(\frac{t^2}{2}(\alpha(\theta_1 L_0 + \theta_2 I_0 - rI_{s_0}) - \mu(\alpha I_{s_0} - \mu R_0))\right) + \dots \end{aligned} \right\} \quad (47)$$

The following initial values of variables are used for the numerical simulation

$$S_0 = 38,178,000, L_0 = 12,000, I_0 = 200,000, I_{s_0} = 3,746, R_0 = 1,045$$

The following parameters values are used for the numerical simulation:

$$\beta = 0.05, \gamma = 0.9, \mu = 0.02, \theta_1 = 0.5, \theta_2 = 0.6, \delta = 0.3, \alpha = 0.6$$

**NUMERICAL SIMULATION AND RESULTS**

The numerical simulation of analytical solution obtained in (47) was carried out using Mathematical software (Maple 18) and the graphical profile of the system responses are presented below

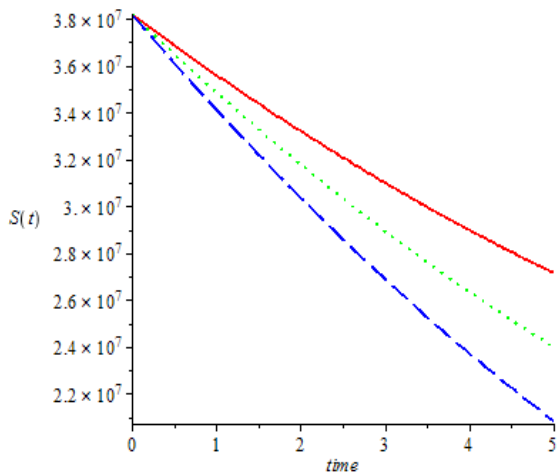


Fig. 4.1: Showing graph of S(t) for different values of  $\beta$  when  $\mu = 0.02, \gamma = 0.9, \delta = 0.3$

Figure 4.1 shows the graphs of  $S(t)$  against  $t$  respectively for different values of  $\beta$ . It was observed that the Susceptible population  $S(t)$  decreases faster as the incidence of Lassa fever  $\beta$  increase

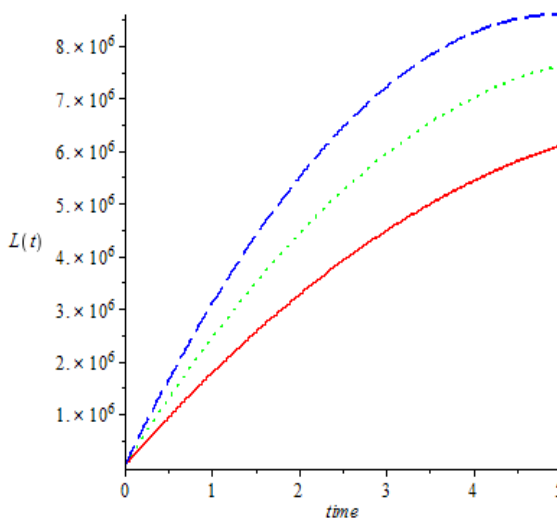


Fig. 4.2: Showing graph of L(t) for different values of  $\beta$  when  $\gamma = 0.9, \delta = 0.3, \mu = 0.02, \theta = 0.5$

Figure 4.2 shows the graphs of  $L(t)$  against  $t$  respectively for different values of  $\beta$ . It was observed that the Latent population  $L(t)$  increases faster as the incidence of Lassa fever  $\beta$  increases.

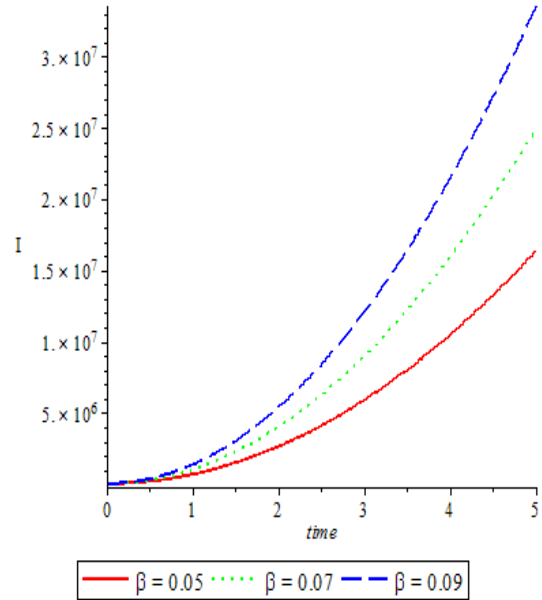


Fig. 4.3: Showing graph of I(t) for different values of  $\beta$  when  $\mu = 0.02, \gamma = 0.9, \theta = 0.5$

Figure 4.3 shows the graphs of  $I(t)$  against  $t$  respectively for different values of  $\beta$ . It was observed that the Infected population  $I(t)$  increases faster as the incidence of Lassa fever  $\beta$  increase

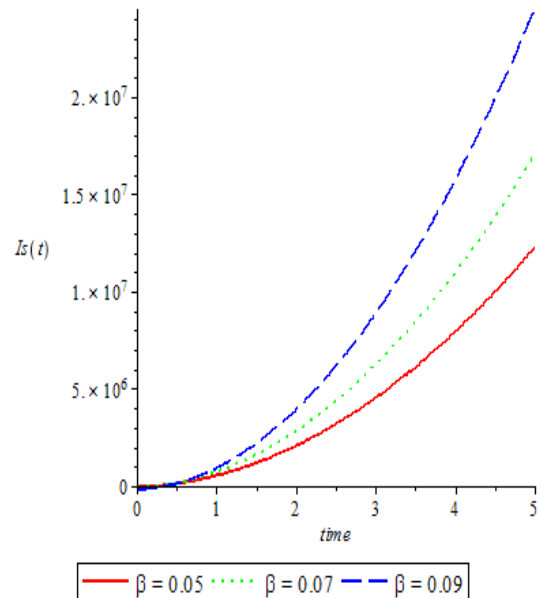


Fig. 4.4: Showing graph of  $I_s(t)$  for different values of  $\beta$  when  $\pi = 0.15, \mu = 0.02, \gamma = 0.9, \delta = 0.3$

Figure 4.4 show the graphs of  $I_s(t)$  against  $t$  respectively for different values of  $\beta$ . It was

observed that the Susceptible population  $I_s(t)$  increases as the incidence of Lassa fever  $\beta$  increases.

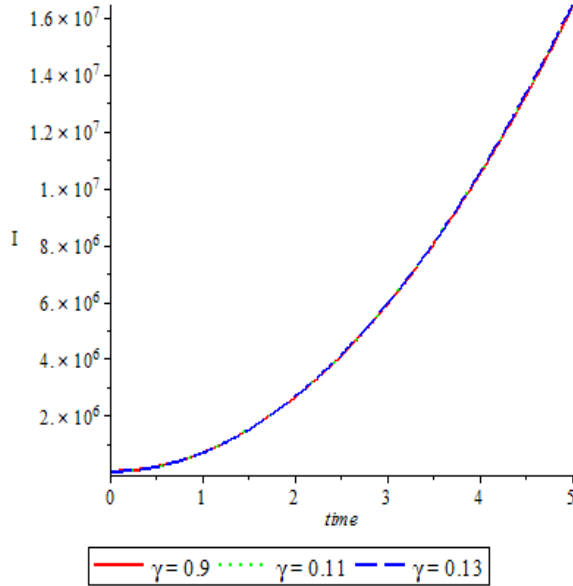


Fig. 4.5: Showing graph of  $I(t)$  for different values of  $\mu = 0.02$ ,  $\beta = 0.05$ ,  $\alpha = 0.6$ ,  $\theta = 0.5$

Figure 4.5 shows the graphs of  $I(t)$  against  $t$  respectively for different values of  $\gamma$ . It was observed that the Infected population  $I(t)$  increases slightly as the Progression rate of individuals to infection class  $\gamma$  increases.

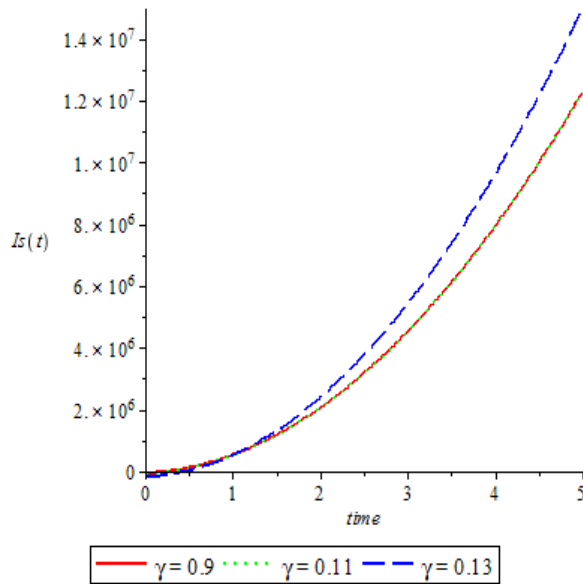


Fig. 4.6: Showing graph of  $I_s(t)$  for different values of  $\gamma$  when  $\mu = 0.02$ ,  $\beta = 0.05$ ,  $\delta = 0.3$ ,  $\theta = 0.5$

Figure 4.6 show the graphs of  $I_s(t)$  against  $t$  respectively for different values of  $\gamma$ . We

observed that as the Isolated class  $I_s(t)$  increases as the rate of individuals to infection class  $\gamma$  increases.

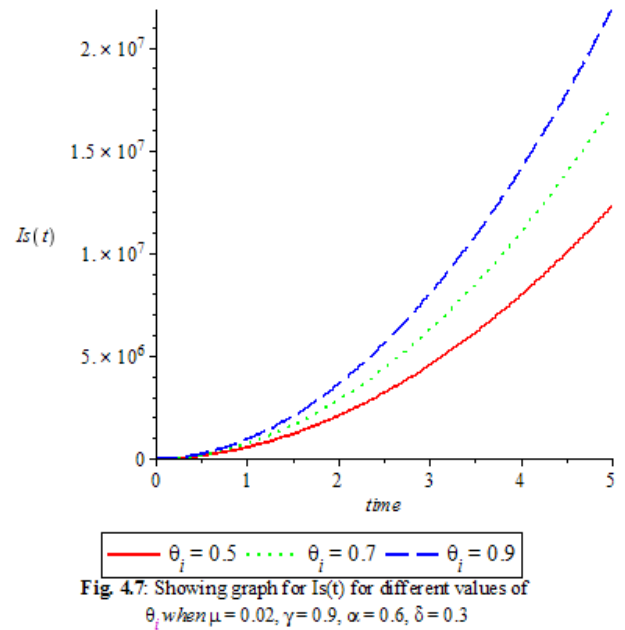


Fig. 4.7: Showing graph for  $I_s(t)$  for different values of  $\theta_i$  when  $\mu = 0.02$ ,  $\gamma = 0.9$ ,  $\alpha = 0.6$ ,  $\delta = 0.3$

Figure 4.7 shows the graphs of  $I_s(t)$  against  $t$  respectively for different values of  $\theta$ . It was observed that Isolated class  $I_s(t)$  increases faster as the rate at which latently infected are isolated due to tracing  $\theta_1$  increases.

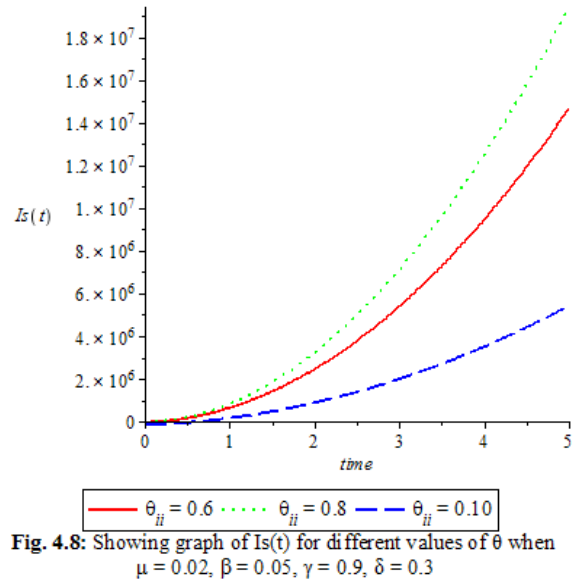


Fig. 4.8: Showing graph of  $I_s(t)$  for different values of  $\theta$  when  $\mu = 0.02$ ,  $\beta = 0.05$ ,  $\gamma = 0.9$ ,  $\delta = 0.3$

Figure 4.8 show the graphs of  $I_s(t)$  against  $t$  respectively for different values of  $\theta$ . It was observed that Isolated class  $I_s(t)$  decreases faster as the rate At which infected individual are isolated  $\theta_2$  increases.

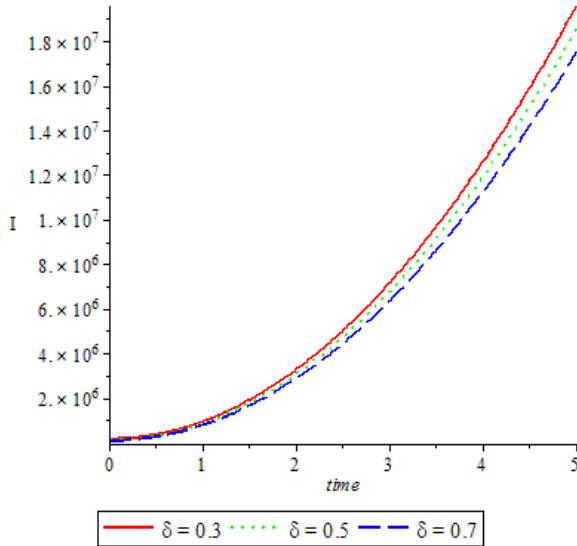


Fig. 4.9: Showing graph of  $I(t)$  for different values of  $\delta$  when  $\mu = 0.02, \gamma = 0.9, \beta = 0.05$

Figure 4.9 shows the graphs of  $I(t)$  against  $t$  respectively for different values of  $\delta$ . It was observed that Infected class  $I(t)$  decreases slightly faster as the Mortality rate due to the disease  $\delta$  increases.

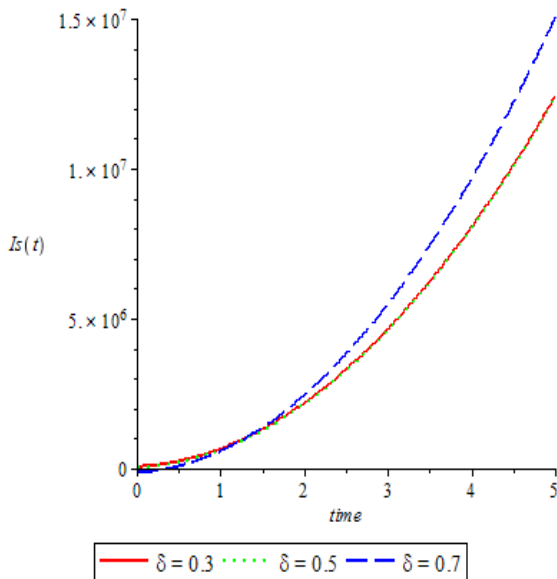


Fig. 4.10: Showing graph of  $I_s(t)$  for different values of  $\delta$  when  $\mu = 0.02, \beta = 0.05, \gamma = 0.9, \theta = 0.5$

Figure 4.10 show the graphs of  $I_s(t)$  against  $t$  respectively for different values of  $\delta$ . It was observed the Isolated class  $I_s(t)$  that undergo treatment increases lightly as the Mortality rate due to the disease  $\delta$  increases.

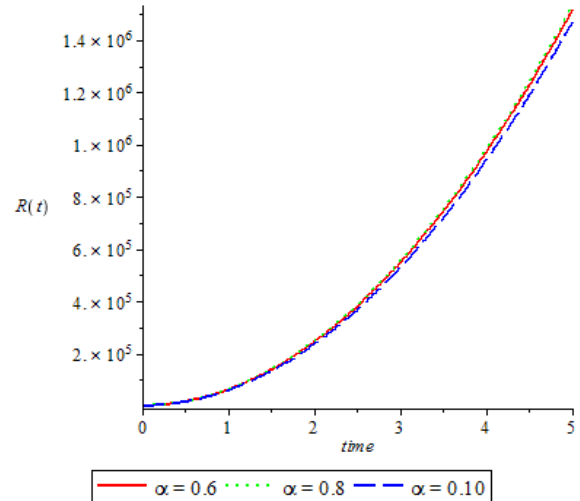


Fig. 4.11: Showing graph of  $R(t)$  for different values of  $\alpha$  when  $\mu = 0.02, \theta = 0.5, \beta = 0.05, \gamma = 0.9, \delta = 0.3$

Figure 4.11 show the graphs of  $R(t)$  against  $t$  respectively for different values of  $\alpha$ . It was observed that as the value of  $\alpha$  increases, the rate of the recovery class  $R(t)$  decreases and as such the rate of the Isolated class  $I_s(t)$  increases.

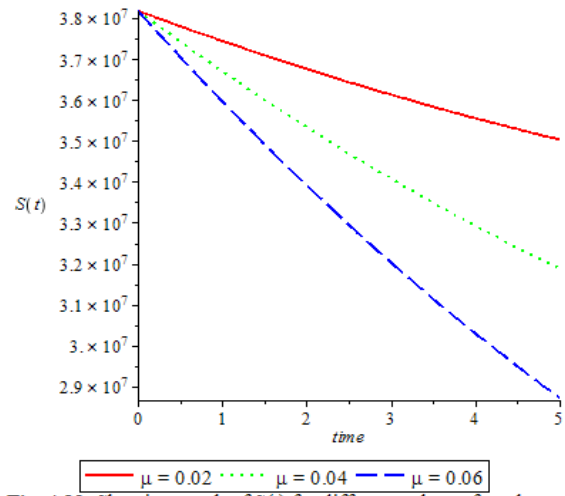


Fig. 4.12: Showing graph of  $S(t)$  for different values of  $\mu$  when  $\gamma = 0.9, \beta = 0.05, \alpha = 0.6, \delta = 0.3$

Figure 4.12 shows the graphs of  $S(t)$  against  $t$  respectively for different values of  $\mu$ . It was observed that Susceptible class  $S(t)$  decreases faster as the natural death rate  $\mu$  increases.

### CONCLUSION

In this study, a deterministic mathematical model was presented to determine the impact of  $\delta$  (death rate due to the disease) on the population dynamics of Lassa fever infection. The model was formalized by a system of Ordinary Differential Equations, and analytical solution of the model was obtained using Homotopy Perturbation Method. The numerical



simulation of the model was carried out using Maple 18 mathematical software. The graphical responses of the system are presented. The results obtained showed that the parameter  $\delta$  introduced into the Isolated class  $I_s(t)$  played a significant role in determining the population size of Infected class  $I(t)$  and Isolated class  $I_s(t)$  at time  $t$ .

The results analysis revealed that more people die due to the disease in the population. This suggest that a treatment class should be introduce so that both the infected class and isolated class can undergo treatment (hospitalization or vaccination) in order to reduce the mortality rate due to the disease in the population

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